Three-Dimensional Thermohydrodynamic Morton Effect Simulation—Part I: Theoretical Model

Introduction

The thermal induced synchronous rotor instability problem, known as the Morton effect, is caused by the journal differential heating in a fluid film bearing. In the case of a synchronous whirl orbit around the equilibrium position, maximum viscous heating occurs around the minimum film thickness area, and this causes a hot spot ($\theta_{\text{HOT}}$) and a cold spot ($\theta_{\text{COLD}}$) leading to a thermal bending ($\theta_{\text{THERM\_BOW}}$) at the bearing location. Convection of heat in the film is known to displace the hot spot ($\theta_{\text{HOT}}$) from $20$ deg to $40$ deg ahead of the high spot ($\theta_{\text{HIGH}}$). The temperature differential across the journal causes a bending moment and resulting bow in the rotating frame that acts like a distributed synchronous excitation in the fixed frame. This thermal bow may cause increased vibration and continued growth of the synchronous orbit into a limit cycle.

In 1973, Tieu [1] formulated a variational principle based on the local potential and a finite element method (FEM) for the evaluation of the 2D fluid film thermal gradient in an infinitely wide bearing. In 1986, Khonsari [2] investigated thermohydrodynamic (THD) effects in the fluid film journal bearing under static load, and developed a computer algorithm to calculate the 2D temperature distribution in the bearing and fluid film utilizing the finite difference method (FDM). Cavitation effect and mixing temperature theory for the evaluation of the inlet temperature were studied. An adiabatic boundary condition on the bearing inner surface was imposed for the computation time reduction.

In 1988, Knight and Barrett [3] presented an approximated method for the THD tilting-pad journal bearing (TPJB) by use of a simplified heat conduction model. Thermal gradient across the film thickness is assumed to have a second-order profile, and pad heat conduction is assumed to be only in the radial direction. The journal surface temperature is calculated by the averaged film temperature in the circumferential direction. In 1990, Earles and Palazzolo [4] calculated a nonlinear transient response of the spinning shaft supported by thermoelastic TPJB excited by a large mass imbalance. It is found that the pad flexibility and thermal effect on fluid film increase vibration level and reduce film thickness producing increased maximum temperature.

In 1993, Keogh and Morton [5] presented an asymmetric viscous shearing in the fluid film journal bearing and this phenomenon is referred to as the Morton effect. They developed a new method to solve the dynamic energy balance problem for a predefined elliptical orbit in the journal bearing. Short bearing theory and iso-viscosity lubricant were assumed for the simplicity. In the energy equation, axial and circumferential conductions were ignored, and 2D temperature in lubricant, bearing, and journal models are considered. The journal surface temperature variations were evaluated by perturbing the orbit. In 1999, Gomiciaga and Keogh [6] presented a new method to evaluate the asymmetric heat flux into the synchronously whirling journal with a prescribed orbit shape based on the orbit time averaged heat flux into the journal and bearing. Steady thermal conditions of the shaft and bearing were evaluated by 3D energy equation adopting finite volume method (FVM) and 3D heat conduction model of the shaft and bearing. In 1999, Larsson [7,8] examined a relation between journal whirling and journal differential heating in the lubricant and shaft interface. In 2004, Balbahadur and Kirk [9,10] developed an analytical method for the Morton effect simulation, and studied a rotor system supported by plain and tilting-pad journal bearings. The results from the theoretical models were compared with the case studies. It was assumed that the hot spot coincides with the high spot, and a cold spot will be formed on the area with maximum film thickness.

In 2010, Murphy and Lorenz [11] developed an approximate and simplified analytical method to evaluate temperature
differential across the journal and its effect on synchronous vibration without the energy equation and shaft heat conduction equation. They conducted the Morton effect simulation adopting a linear vector relationship where each vector element has an amplitude and phase. It is assumed that the thermal response is infinitely slow compared with the rotor-dynamic response. In 2012, Childs and Saha [12] presented a computational algorithm for the Morton effect. The shaft thermal gradient induced by the initial elliptic orbit was solved by adding the temperature distributions for the separate forward and backward orbits. Reducing fluid film viscosity and the overhung mass were suggested to mitigate the Morton effect problem.

In 2013, Palazzolo and Lee [13] suggested a finite element (FE) model for the varying viscosity fluid film and thermal shaft model. The elastic rotor model was analyzed, combined with the THD tilting-pad bearing, and transient analysis was done adopting a staggered integration scheme for the computation time reduction. An insulated thermal boundary condition was imposed at the contact between fluid film and bearing liner, that means the pad heat conduction and the resultant pad thermal expansion are not taken into account. A 2D energy equation was used for the lubricant temperature evaluation.

To predict the Morton effect problem without a loss of accuracy, a more sophisticated numerical modeling is needed. This research is an extension of Palazzolo and Lee’s [13] earlier work. This study adopted a more rigorous analysis model such as: (1) 3D energy equation, (2) 3D heat conduction model of bearing and shaft, (3) various thermal boundary conditions, (4) asymmetric shaft and pad thermal expansion in the circumferential and axial direction calculated by the FE model, and (5) nonlinear time transient dynamic and thermal analysis with a variable time step numerical integration scheme.

Earlier studies evaluated thermal bow angle and phase by analytical method [13]; however, in this research, shaft thermal bending and expansion was evaluated by the 3D thermal gradient in the FE shaft model. Earlier studies assumed that the lubricant temperature gradient in the axial direction has a uniform profile; however, in the case of Morton effect simulation with bigger orbit size, 3D energy is found to be required for the reliable prediction. The simulation results by 2D and 3D energy equations will be compared by the static equilibrium analysis with zero orbit and rotating orbit. The modified mixing temperature theory was adopted for the computation time reduction without losing accuracy. Thermal time constants of the shaft and bearing structure are much higher than for vibration; therefore, the thermodynamic coupled transient analysis model requires a long computation time. So, a staggered integration scheme [13] is adopted to increase the computation efficiency without a loss of accuracy of the dynamic and thermal transient models.

1 Thermoelastodynamic Rotor–Bearing Model

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1.1 Dynamic Rotor Model. Lateral vibration is considered for the dynamic analysis of the rotor system, and modal coordinate transformation is used for the computation time reduction. Modal analysis is a useful tool for the rotor–bearing system analysis for the reduction of degrees of freedom without a loss of accuracy. The system response can be assumed to be a linear combination of the eigenvectors which are orthogonal. The equation of motion for the rotor–bearing system in a physical coordinate can be expressed by

\[ \mathbf{M}_R \ddot{\mathbf{X}}_R + (\mathbf{C}_\text{GYRO} + \mathbf{C}_\text{BRG}) \dot{\mathbf{X}}_R + (\mathbf{K}_\text{STRUCT} + \mathbf{K}_\text{BRG}) \mathbf{X}_R = \mathbf{F}_R \]  

The state vector \( \mathbf{z}_R = \begin{bmatrix} \mathbf{X}_R \mathbf{X}'_R \end{bmatrix}^T \) allows the second-order matrix-vector differential equations to become

\[ \begin{bmatrix} \mathbf{X}_R \\dot{\mathbf{X}}_R \end{bmatrix} = \begin{bmatrix} -\mathbf{M}_R \mathbf{C}_R & -\mathbf{M}_R \mathbf{K}_R \\ \mathbf{I} & 0 \end{bmatrix} \mathbf{X}_R + \begin{bmatrix} \mathbf{M}_R \mathbf{F}_R \end{bmatrix} \]  

where \( \mathbf{K}_R = \mathbf{K}_\text{STRUCT} + \mathbf{K}_\text{BRG} \) and \( \mathbf{C}_R = \mathbf{C}_\text{GYRO} + \mathbf{C}_\text{BRG} \). This equation can be shortened as

\[ \dot{\mathbf{z}}_R = \mathbf{D}_R \mathbf{z}_R + \mathbf{\tilde{F}}_R \]  

If the free vibration problem is considered by setting \( \mathbf{\tilde{F}}_R = 0 \), and \( \mathbf{z}_R = \psi_R e^{i \omega t} \) that yields

\[ \lambda_i \psi_R = \mathbf{D}_R \psi_R, \]  

The eigenvalues of \( \mathbf{D}_R \) are the same as the eigenvalues of \( \mathbf{D}_0 \); however, the eigenvectors of \( \mathbf{D}_R \) are not identical to the ones of \( \mathbf{D}_0 \). The eigenvectors of \( \mathbf{D}_R \) satisfy

\[ \lambda_i \psi_R = \mathbf{D}_R \psi_R, \]  

Typically, \( \psi_R \) is called a right eigenvector, and \( \phi_R \) is referred to as a left eigenvector of \( \mathbf{D}_0 \). If \( \psi_R \) is normalized so that \( \phi_R^T \psi_R = 1 \), \( \psi_R \) and \( \phi_R \) satisfy the following conditions [14]:

\[ \phi_{Rm}^T \phi_{Rn} = \begin{cases} 0, & \text{if } m \neq n \\ 1, & \text{if } m = n \end{cases} \]  

\[ \phi_{Rm}^T \phi_{Rn} = \begin{cases} 0, & \text{if } m \neq n \\ \lambda_i, & \text{if } m = n \end{cases} \]  

Hence, the utilization of left and right eigenvectors diagonalizes the first-order matrix-vector differential equation. Since only a small number (L) of the entire eigenmodes (N) are used to evaluate the linear combination of eigenvectors, the approximate expression for \( \mathbf{z}_R \) can be expressed by

\[ \mathbf{z}_R(t) = \sum_{l=1}^{L} \xi_l(t) \psi_R l \quad (L \ll N) \]  

In this research, the rotor dynamic behavior is assumed to be dominated by lower eigenmodes, and the highest eigenmode is chosen at three times as high as the rotor spin frequency.

1.2 Dynamic Pad Model With Flexible Pivot. The pad dynamic model in this research is capable of the pad tilting and pitch motions, and the pivot translation motion. The pitch motion is considered only in the spherical pivot model. The pivot flexibility is known to lower the rotor’s critical speeds, and increase the vibration amplitude due to the decreased damping coefficients. The equation of pad motion is derived from the Lagrange equation [15], and it is described in Eqs. (9) and (10), where the pad pitch motion is not explained. In the case of the spherical pivot, one more degree of freedom is considered, and the pad tilting and pitch motions are assumed to be independent of each other. Figure 1 shows the two degree of freedom pad dynamic model, where both pad tilting and translation motions are considered. The fluidic force \( \mathbf{F}_o(t) \) acting on the pad, and the fluidic moment \( \mathbf{M}_o(t) \) acting on the pivot are evaluated by the generalized Reynolds equation at each time step. Pad translational motion in the circumferential direction (x) is ignored. The derived pad nonlinear...
dynamic equation is used for the time transient analysis of the THD rotor–bearing system:

\[ mCGp_{pt} - e_{G}mCG\delta_{iul}\cos(\delta_{iul}) + e_{G}mCG\delta_{iul}^2 \sin(\delta_{iul}) + K_{p}\rho_{pt} = F_{o}(t) \]  \hspace{1cm} (9)

\[ J_{G}\delta_{iul} + e_{G}mCG\cos(\delta_{iul})\rho_{pt} + e_{G}mCG\delta_{iul}^2 \sin(\delta_{iul}) - e_{G}mCG\delta_{iul}^2 \sin(\delta_{iul})\cos(\delta_{iul}) = M_{G}(t) \]  \hspace{1cm} (10)

1.3 Heat Conduction Model. If a more accurate solution of the thermal bow causing a radial thermal gradient in the shaft is desired, the axial temperature distribution in the rotor must be included in the model. In other words, the rotor length should be considered in the shaft heat conduction model. However, the consideration of the full length will take a considerable amount of computation time for the size of the FE matrix. The ultimate object of the shaft heat conduction model is the accurate prediction of the shaft thermal bending at the bearing location. For this reason, seven times the entire bearing length is considered as a whole thermal shaft length in the model. If the journal surface, diametral temperature differentials at both ends of the thermal shaft decay into zero, the length of the thermal shaft model is enough for the accurate thermal bow prediction. Thermal behavior in the shaft is governed by Laplace equation. Time independent, constant thermal conductivity is assumed, and the heat flow governing equation goes as follows:

\[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{\rho c}{k} \frac{\partial T}{\partial t} \]  \hspace{1cm} (11)

Three types of thermal boundary conditions can be imposed on the system and are shown as

\[ T = T^* \quad \text{on } S_{Tpt} \]
\[ q = q^* \quad \text{on } S_{Tgt} \]
\[ q = h(T - T^*) \quad \text{on } S_{Tfc} \]  \hspace{1cm} (12)

On the boundary \( S_{Tpt} \), the temperature is prescribed (\( T^* \)), and the convective boundary condition is imposed on \( S_{Tfc} \) with the ambient temperature \( T^* \) and convective coefficient \( h \). In the boundary \( S_{gpt} \), heat flux is prescribed at \( q^* \). The boundary is thermally insulated if there is no defined boundary condition. Thermal behavior in the pad is also governed by the Laplace equation.

The transient thermal equation (11) is solved by the FE method [16] utilizing a 3D eight node isoparametric element to discretize the model domain as seen in Eq. (13), where \([F]\) is a thermal loads vector that varies with time. If \([C]\) and \([K]\) in Eq. (13) are assumed to be time independent, a time-varying solution can be obtained by the modal coordinate transformation yielding a greatly reduced computation time [13]. The temperature solution is assumed to be dominated by lower eigenmodes, and temperature distribution is assumed to be a smooth shape. The eigenvalues and eigenvectors can be obtained from Eq. (14) where \([T]\) is an eigenvector. If each eigenvector \([T]_i\) is normalized with respect to \([C]\) this means \( [C]^T[C][T] = [I] \) and \( [K]^T[K][T] = [\lambda] \). If we let \([\phi]\) be a modal matrix, physical temperature is transformed to the generalized modal temperature (\([\bar{Q}]\)) as seen in Eq. (15):

\[ [C][\bar{T}] + [K][\bar{T}] = [F] \]  \hspace{1cm} (13)

\[ ([K] - \lambda[C])[\bar{T}] = 0 \]  \hspace{1cm} (14)

\[ [\bar{T}] = [\phi][Q] \]  \hspace{1cm} (15)

The above calculation yields the following uncoupled heat conduction equation whose degree of freedoms is reduced leading to a greatly reduced computation time

\[ [\bar{Q}] + [\lambda][\bar{Q}] = [F] \]  \hspace{1cm} (16)

1.4 Thermal Distortion Model. Thermal deformation of the shaft and pad is calculated by the 3D FE model, and the modal information is shared with the shaft and pad heat conduction model discussed in Sec. 1.3. The deformation of the elastic shaft and pad can be described by means of the equilibrium equation and constitutive equation. The deformation vector is composed of elastic deformation and thermal induced distortion, and the superposition principle is used to add to the two deformation effects. The effect of shaft thermal expansion seen in Fig. 2(a) can be divided into two separate effects on the nonlinear transient rotor-dynamic analysis. First, the asymmetric journal heating causes thermal bow as seen in Fig. 2(b), that is a well-known phenomenon to other researchers, and the second effect is an asymmetric thermal expansion of the shaft in both the axial and circumferential direction as seen in Fig. 2(c). The current 3D elastic FE shaft model is capable of predicting both the asymmetric thermal expansion and bow.

1.5 Film Thickness Evaluation With 3D Pad and Shaft Motion. In this study, pad tilting and pitch, and journal translation, pitch, and yaw motions are taken into account to simulate the rotor–bearing 3D motion. Most former researches have considered only journal and pad 2D motion in the \( x-y \) plane as seen in Fig. 3(a), which means the shaft’s translation motion and pad tilting and translation motions in the \( x-y \) plane are simulated. A 2D motion film thickness formula is modified to consider the shaft
pitch ($\theta_p$) and yaw ($\theta_y$) motions, and pad pitch motion ($z_{pitch}$) as seen in Eq. (12). The pad pitch motion is taken into account only in the case of the spherical pivot model as seen in Fig. 3(b). The temporal derivative of the film thickness formula is given by (18). Thermal expansion of the spinning journal $[h_{TEJ}(t, \theta, z)]$ and pad $[h_{TEP}(\theta, z)]$ are also taken into account in the film thickness formula, and temporal derivative of the spinning journal thermal expansion observed in the fixed frame $[dh_{TEJ}(t, \theta, z)/dt]$ is taken into account. This effect will be discussed in depth in Sec. 1.6.

$$h(\theta, z) = CL_F - [\varepsilon + z\theta_y - p_{pot}\cos(\theta_p) - z_{pitch}\cos(\theta_p)]\cos(\theta)$$
$$- [\varepsilon + z\theta_y - p_{pot}\sin(\theta) - z_{pitch}\sin(\theta_p)]\sin(\theta)$$
$$- (CL_F - CL_B)\cos(\theta)$$
$$- \theta_{thf}R\sin(\theta - \theta_p)$$
$$- h_{TEJ}(t, \theta, z) = h_{TEP}(\theta, z)$$
(17)

$$\frac{dh(\theta, z)}{dt} = - \left( \frac{d\varepsilon}{dt} + z\frac{d\theta_y}{dt} - \frac{dp_{pot}}{dt}\cos(\theta_p) - z_{pitch}\frac{d\cos(\theta_p)}{dt} \right)\cos(\theta)$$
$$- \left( \frac{d\varepsilon}{dt} + z\frac{d\theta_y}{dt} - \frac{dp_{pot}}{dt}\sin(\theta) - z_{pitch}\frac{d\sin(\theta_p)}{dt} \right)\sin(\theta)$$
$$- \frac{d\theta_{thf}}{dt}R\sin(\theta - \theta_p) - \frac{dh_{TEJ}(t, \theta, z)}{dt}$$
(18)

### 1.6 Asymmetric Thermal Expansion of the Shaft

Prior studies ignored the effect of asymmetric thermal expansion of the shaft surface caused by the journal surface, diametral temperature differential. Earlier studies had considered uniform thermal expansion of the shaft surface both in the axial and circumferential directions, where the volume averaged temperature and other simplified assumptions were used. In this section, a new shaft thermal expansion model is presented, where the 3D thermal gradient inside the 3D thermoelastic shaft is considered.

If the journal diametral temperature differential exists, the asymmetric thermal expansion will also exist as well. Figure 4(a) shows the spinning ($\omega \text{ rad/s}$) shaft with the nonuniform thermal expansion in the circumferential direction ($\theta$). Note that $h_{TEJ}(t, \theta, z)$ is defined as the amount of the thermal expansion of the journal surface in the radial direction ($r$) due to the asymmetric journal heating, and observed at the fixed frame. So this is a function of the time ($t$), circumferential ($\theta$), and axial ($z$) positions as seen in Figs. 4(a) and 4(b).

The amount of the film thickness reduction due to the thermal expansion of the spinning journal is $h_{TEJ}(t, \theta, z)$ as seen in (19). The shift of the center of mass due to the thermal expansion is assumed to be zero, that means that the shaft spins on the original $z$ axis. This asymmetric thermal deformation of the spinning journal makes a change to the temporal derivative of the film thickness reduction ($dh_{TEJ}/dt$) as well as the film thickness reduction ($h_{TEJ}$). Figure 4(b) shows the schematic diagram for the evaluation of $dh_{TEJ}/dt$. Point A indicates a specific nodal position on the journal surface before the thermal expansion. The thermal expansion of the journal in the radial direction ($r$) shifted point A to B. Note that the slope ($d\theta$) at point B is defined as the rate at which the amount of the thermal expansion ($h_{TEJ}$) changes in the circumferential direction ($\theta$), and measured at a fixed frame as seen in (20).

The slope ($d\theta$) will be zero at the hot spot and the cold spot, and have the maximum value between the two spots if the journal surface temperature changes sinusoidally in the circumferential direction. The linear velocity at point B ($v_B$) can be defined by (21).

The decreasing rate of the film thickness at point B due to the asymmetric shaft thermal expansion ($dh_{TEJ}/dt$) can be evaluated by multiplying the slope ($d\theta$) by the linear velocity ($v_B$) as seen in (22). And the time derivative of the film thickness ($dh/dt$) can be evaluated by subtracting the decreasing rate from the original temporal derivative of the film thickness as seen in Eq. (23). The slope in the axial direction does not affect the time derivative of the film thickness ($dh/dt$) because the shaft surface is assumed to move only in the circumferential direction ($\theta$).

Note that the film thickness ($h$) and the temporal derivative of the film thickness ($dh/dt$) are evaluated at a fixed frame, and they are updated each time step of the time transient THD analysis for the evaluation of the fluidic force acting on the pad and rotor, and the moment acting on the pivot.

Decrease of the film thickness due to the thermal expansion of the pad ($h_{TEP}$) is also taken into account. Both the lubricant and pad are on a fixed frame, that means both elements are assumed to be fixed in the circumferential direction ($\theta$).

The thermal gradient in the pad and shaft is assumed to be quasi-static during the dynamic transient analysis. So the temporal derivative of the film thickness due to the pad thermal distortion ($dh_{TEP}/dt$) is assumed to be zero. In reality, it would be negligible:

$$h_{TEJ}(t, \theta, z) = h_{TEJ}(t, \theta, z) + h_{TEP}(\theta, z)$$
(19)
1.7 Thermal Bow Induced Imbalance Force. The thermal bending eccentricity and phase caused by the asymmetric journal heating is known to cause a significant change to the rotor-dynamic behavior. In earlier studies [9,10,12,13] thermal bow angle and phase were evaluated by an analytical method utilizing a journal diametral temperature differential and a hot spot phase, where the phase differential between the hot spot and the thermal bow is assumed to stay constant at 180 deg. and the thermal gradient between the hot spot and the cold spot is linear. Figure 5(c) shows the phase relationship between the hot spot, cold spot, high spot, heavy spot, and thermal bow. If the journal surface temperature shows a sinusoidal variation in the circumferential direction, and the phase differential between the hot spot and cold spot stays constant at 180 deg, then the thermal bow phase will coincide with the cold spot phase. This phase relationship has been assumed in earlier studies. This study does not use the assumed phase relationship but use a 3D energy equation and 3D thermal gradient inside the 3D thermoelastic FE shaft model to calculate the thermal induced bending phase ($\theta_{n}^{TB}$) and the eccentricity ($e$) which are shown in Fig. 5.

In earlier studies, a simply assumed thermal induced imbalance force acting on a lumped overhung mass has been taken into account. In reality, the thermal induced imbalance force does not act on a single lumped mass but is distributed in the rotor length since the thermal bow effect is continuous in the axial direction. This research presents a new method for the evaluation of the thermal bending eccentricity, phase, and the resultant thermal induced imbalance forces distributed on the nodes ($n$) of the FE rotor-dynamic model as shown in Eqs. (26) and (27).

First, the thermal bow induced eccentricity and the phase should be calculated to produce the thermal induced imbalance force seen in Eqs. (26) and (27). Shaft thermal distortion is evaluated by use of the 3D elastic FE model, and the shaft thermal gradient is calculated by the heat conduction FE model. Figures 5(a) and 5(b) show the schematic diagram for the evaluation of the thermal bending induced eccentricity distribution. The symbol $\xi$ denotes the thermal bow axis lying on the x-y plane. In this study, the simulation model has a single overhung mass located on the right side of the rotor system as seen in Fig. 10. So the thermal induced eccentricity distribution is considered on the right hand side of the NDE bearing location ($z_{NDE}$), which means that the thermal imbalance force on the left side of the NDE bearing is assumed to be zero [see the first case of Eq. (24)]. If the axial position of the dynamic rotor node is located inside the thermal shaft [$z_{NDE} < z < l_{r}$ in Fig. 5(a)], the nodal eccentricity value ($e_{n}$) is evaluated by the eccentricity ($e_{F}$) calculated by the thermal deformation of the shaft FE model. Since the dynamic rotor node and the thermal shaft nodal position in the axial direction do not coincide, the interpolation method is used [see the second case of Eq. (24)]. If the axial position of the dynamic rotor node is outside the thermal shaft [$z > l_{r}$ in Fig. 5(a)], then the thermal bending angle ($\beta_{n}$), the eccentricity ($e_{n}$) at the end of the thermal shaft, and the distance ($d_{n}$) between the right end of the thermal shaft and the current node’s axial position are used for the evaluation of the thermal bow induced eccentricity ($e_{n}$) [see the third case of Eq. (24)]. The nodal thermal bow phase distribution at nth node ($\theta_{n}^{TB}$) is evaluated by the same method as in the calculation of the thermal eccentricity [see Eq. (25)]. The thermal bow induced imbalance force ($F_{mx}, F_{my}$) can be expressed by means of nodal values acting on the FE rotor-dynamic model as seen in Eqs. (26) and (27), where $M_{n}$ denotes a point mass of nth node of the FE rotor-dynamic model, and $N_{ROTOR}$ indicates the number of nodes of the rotor-dynamic FE model:

$$e_{n} = \begin{cases} 0, & 0 \leq z < z_{NDE} \\ e_{F}, & z_{NDE} \leq z < l_{r} \\ e_{n} + d_{n} \sin(\beta_{n}), & l_{r} \leq z \end{cases}$$  (24)
2 Fluid Film Model

2.1 Generalized Reynolds Equation. The governing equation to obtain a pressure distribution in the thin film is the Reynolds equation, which models the thin fluid film between the two moving planes. The thin film is denoted as the fluid film thickness and is represented by \( h \). The Reynolds equation can be derived from the momentum and the continuity equations [17], yielding pressure and velocity distribution. Integration of the momentum equation provides the velocity distribution in terms of pressure gradients. Substitution of this velocity profile into the flow rate continuity equation will give the Reynolds equation. The generalized Reynolds equation can be expressed as

\[
V(D_1 \nabla p) + (V D_2) (U_2 - U_1) + (\nabla h) U + \frac{\partial h}{\partial t} = 0
\]  

(28)

The above generalized Reynolds equation can be reduced to the following for a fixed bearing and spinning shaft:

\[
V(D_1 \nabla p) + (V D_2) U + \frac{\partial h}{\partial t} = 0
\]  

(29)

For a thermohydrodynamic problem, viscosity is a function of the temperature, and the constants \( D_1 \) and \( D_2 \) can be expressed by

\[
D_1 = \int_0^h \int_0^l \frac{\partial}{\partial z} \frac{\partial}{\partial z} dz - \int_0^h \int_0^l \frac{\partial}{\partial z} \frac{\partial}{\partial z} dz = \int_0^h \int_0^l \frac{\partial}{\partial z} \frac{\partial}{\partial z} dz
\]  

(30)

\[
D_2 = \int_0^h \int_0^l \frac{\partial}{\partial z} \frac{\partial}{\partial z} dz
\]  

(31)

The fluid film in the generalized Reynolds equation is assumed to have a laminar flow, invariant pressure in the film thickness direction, a negligible shaft curvature effect, negligible fluid inertia, constant fluid density, and temperature-affected viscosity. Additionally, it is assumed to be an incompressible Newtonian fluid and nonslip at the solid and fluid interface. The relationship between viscosity and temperature of the fluid can be expressed as

\[
\mu = \mu_0 e^{-\beta(T - T_0)}
\]  

(32)

This research considers Reynolds cavitation boundary condition employing a back substitution procedure suggested by Lund and Thomsen [18].

2.2 Three-Dimensional Velocity Profile. From the Reynolds equation, the velocity profile can be evaluated by the following equation [19]:

\[
u = \left( \int_0^l \int_0^z \frac{\partial}{\partial z} \frac{\partial}{\partial z} dz - \int_0^l \int_0^z \frac{\partial}{\partial z} \frac{\partial}{\partial z} dz \right) \nabla p + \frac{\int_0^l \int_0^z \frac{\partial}{\partial z} \frac{\partial}{\partial z} dz - \int_0^l \int_0^z \frac{\partial}{\partial z} \frac{\partial}{\partial z} dz}{\int_0^l \int_0^z \frac{\partial}{\partial z} \frac{\partial}{\partial z} dz} U
\]  

(33)

In a conventional 2D analysis, the velocity profile is obtained only at the bearing axial center; however, for the full 3D bearing and fluid film model, the velocity profile should also be obtained in the axial direction. In the case of 2D analysis, \( \nabla p \) in Eq. (33) was only in the circumferential direction; however, in the 3D analysis, \( \nabla p \) is evaluated both in the circumferential and axial direction. Thus, the velocity profile \( u \) becomes a 3D vector.

2.3 Three-Dimensional Energy Equation. The temperature distribution in the thin fluid film is governed by the energy equation, which needs thermal boundary conditions, pressure, velocity, and viscosity distribution. The Reynolds equation and the energy equation are coupled via the viscosity profile. The updated viscosity distribution is used in solving the Reynolds equation. For a laminar flow, incompressible, and Newtonian fluid, the energy equation is expressed by [13]

\[
pc \left( \frac{\partial T}{\partial t} + \frac{\partial w}{\partial z} \frac{\partial T}{\partial z} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]  

(34)

where the conduction is considered in the \( x, y, \) and \( z \) directions, and the convection term is only considered in the \( x \) and \( z \) directions since the velocity profile created by the Reynolds equation has only \( x \) and \( z \) components. The dissipation term is only considered by \( \frac{\partial}{\partial z} \frac{\partial}{\partial z} \) term since the thin film thickness is only in the film thickness direction. According to Gadangi’s research [20], there was no noticeable difference between transient and steady state energy equations in large imbalance transient analysis. So time dependent lubricant temperature \( \frac{\partial T}{\partial t} \) is not taken into account as seen in Eq. (34).

Thin film temperature has a strong influence on the viscosity distribution, and the pressure distribution does not. Viscosity can be assumed to follow an exponential relationship given by Eq. (32). The finite element method is used for solving an energy equation. Eight node isoparametric elements are used for discretizing the film domain. The solution of Eq. (34) exhibits spatial temperature oscillation problems as a result of the convection term on the left hand side of Eq. (34), so the quadratic up-winding scheme proposed by Zienkiewicz et al. [21] is employed to mitigate the problem. The boundary conditions of the energy equation are as follows:

(a) The one-dimensional mixing temperature theory in the axial direction is used for the inlet fluid temperature.
(b) For the cavitation region, the dissipation term in the energy equation is set to zero.
(c) When cavitation occurs, the lubricant thermal conductivity is replaced with an air thermal conductivity.

2.4 Heat Flux Boundary Condition. The heat flux and temperature boundary condition assigned to the interface between the lubricant and bearing pad can be expressed by Eqs. (35) and (36).

On the other hand, the heat flux boundary condition between the journal and lubricant is not simple due to the moving frame of the spinning shaft and the fixed frame of the lubricant. From the shaft’s frame spinning counterclockwise (CCW), the lubricant’s fixed frame is moving in a clockwise (CW) direction as seen in Fig. 6. So the flux boundary condition can be expressed by Eqs. (37) and (38). If the journal is moved into \( \theta = \omega t \) in the CCW direction, the lubricant’s angular position that meets the shaft’s position of \( \theta = 0 \) is \( \theta = \omega t \). At each time step, the heat flux boundary condition between lubricant and spinning shaft is saved in the computer’s memory. After a single orbit, the orbit time averaged boundary condition is used for the thermal boundary condition of the spinning shaft. For the heat flux boundary...
conservation: the following relationship can be established according to the energy of the mixing temperature theory.

The fluid temperature distribution flowing into the leading edge is a function of the lubricant temperature. For the evaluation of the inlet fluid temperature, a modified boundary condition of the lubricant in a fixed frame, a modified boundary condition is adopted as seen in Eqs. (39) and (40):

\[ T_{\text{in}} = \frac{Q_{\text{in}}^{-1}}{Q_{\text{supply}}} \left( T_{\text{in}}^{-1}(z) + \frac{Q_{\text{in}}(z) - Q_{\text{out}}^{-1}(z)}{T_{\text{in}}^{-1}(z) + Q_{\text{supply}}(z)} \right) \]

where \( i \) denotes the pad number.

In some cases, \( Q_{\text{out}} \) may exceed \( Q_{\text{in}} \), which means \( Q_{\text{in}} - Q_{\text{out}} < 0 \), and the conventional mixing temperature theory in Eq. (43) predicts a physically wrong inlet temperature, so there should be a compensation term to prevent this problem.

Another physically impossible case occurs when \( Q_{\text{in}} - Q_{\text{out}} = 0 \), which means there is no supply fluid temperature effect in the inlet temperature, which means all recirculated fluid flow into the next pad, and no supply fluid goes into the leading edge of the pad. So, a mixing coefficient in Eq. (44). If \( Q_{\text{out}} \) becomes larger than \( \eta Q_{\text{in}} \), the recirculated flow is limited to \( \eta Q_{\text{in}} \). And if \( Q_{\text{out}} \) is smaller than \( \eta Q_{\text{in}} \), the conventional mixing equation (43) is used. In this study, \( \eta \) is fixed at 0.8.

\[ T_{\text{in}} = \frac{Q_{\text{in}}^{-1}}{Q_{\text{supply}}} \left( T_{\text{in}}^{-1}(z) + \frac{Q_{\text{in}}(z) - Q_{\text{out}}^{-1}(z)}{T_{\text{in}}^{-1}(z) + Q_{\text{supply}}(z)} \right) \]

where \( i \) denotes the pad number.

2.5 Mixing Temperature Theory With the 3D Bearing Model. The fluid temperature distribution flowing into the leading edge of the pad has a significant effect on the bearing dynamic behavior since the fluidic force is influenced by the viscosity, which is a function of the lubricant temperature.

For the evaluation of the inlet fluid temperature, a modified mixing temperature theory is introduced. The amount of the flow into the pad leading edge becomes the sum of the supply and upstream flow, and the temperature difference between the upstream flow and supplying flow makes the following derivation of the mixing temperature theory.

The temperature and the amount of the supply flow are defined as \( T_{\text{supply}} \) and \( Q_{\text{supply}} \), respectively, and the temperature and the amount of recirculated flow are \( T_{\text{out}} \) and \( Q_{\text{out}} \). The temperature and amount of flow entering into the leading edge are defined as \( T_{\text{in}} \) and \( Q_{\text{in}} \). The schematic diagram for mixing theory is drawn in Fig. 7. Since the conservation of fluid mass can be expressed by

\[ Q_{\text{in}} = Q_{\text{out}} + Q_{\text{supply}} \]  

the energy in the oil groove volume should be conserved, and if all the mixed fluid is assumed to go into the downstream pad, the following relationship can be established according to the energy conservation:

\[ Q_{\text{in}} T_{\text{in}} = Q_{\text{out}} T_{\text{out}} + Q_{\text{supply}} T_{\text{supply}} \]  

Substituting Eq. (42) into Eq. (41), the temperature boundary condition at the leading edge is evaluated by mixing temperature theory as seen by

\[ T_{\text{in}} = \frac{Q_{\text{in}}^{-1}}{Q_{\text{supply}}} \left( T_{\text{in}}^{-1}(z) + \frac{Q_{\text{in}}(z) - Q_{\text{out}}^{-1}(z)}{T_{\text{in}}^{-1}(z) + Q_{\text{supply}}(z)} \right) \]

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staggered integration scheme assumes that once the steady state of the THD system with a smaller thermal time constant is achieved, there will be no significant change in the system without changing the thermal boundary conditions surrounded by the bearing and the spinning shaft.

After the THD system reaches the steady state, a new heat flux boundary condition is reevaluated, and it is applied into the initial condition of the thermal-only system. After a relatively longer simulation time of the thermal-only system, the final temperature distribution of the bearing and rotor is used for the new thermal boundary conditions of the THD system. This alternate process is extended to get the transient analysis results. The time span of the thermal system’s integration is decided by a trial and error. The algorithm for the Morton effect simulation is shown in Fig. 8, and the following explains it:

(1) Temperature distribution in the shaft (Yo_THERM_shaft) and pads (Yo_THERM_pads), and displacement and velocity of the flexible rotor and rigid pads (Yo_0) become the initial conditions of the Morton effect analysis.

(2) sub_DYN_Transient is for a variable time step numerical integration of THD analysis. The dynamic equation is integrated from t0 to tE_dyn with the initial state Yo_DYN. There are two types of imbalance forces. The first one is the initial mechanical imbalance force, and the second one is the thermal bow induced imbalance force. The initial mechanical eccentricity stays constant with time, and the thermal induced imbalance is recalculated at each staggered integration time step based on the thermal gradient and the resultant thermal distortion of the shaft. During the dynamic transient analysis, six subprocedures are conducted as follows:

(2.1) sub_Filmthickness evaluates the film thickness (h) and time derivatives of the film thickness (dh/dt) given the rotor and pads states (Yo_DYN). Thermal expansion of shaft (ΔT_shaft) and pad (ΔT_pad) are taken into account in the calculation of h and dh/dt, which is explained in Sec. 1.6.

(2.2) Based on h and dh/dt, fluidic force and velocity profiles are calculated by sub_REYNOLDS, and the velocity profile (VFluid) is used for the evaluation of the lubricant temperature (TempFilm) and viscosity (μ), which makes a significant effect on the fluidic force (FFluid).

(2.3) Fluidic force (FFluid), gravity force (FGravity), initial mechanical imbalance force (FME), and thermal induced imbalance force (FThermal) are imposed to the dynamic transient analysis of the rotor-bearing system. The net imbalance is calculated as a vector sum of initial imbalance and the thermal induced imbalance.

(2.4) sub_Mixing evaluates the lubricant temperature distribution flowing into the leading edge of each pad, using the one-dimensional mixing temperature theory discussed in Sec. 2.5.

(2.5) 3D energy equation is adopted for the evaluation of the temperature distribution (TempFilm) in the fluid film considering axial convection and conduction. Viscosity distribution (μ), that is a function of the lubricant temperature (TempFilm), is updated in the energy equation. Inlet temperature (TempInlet), shaft temperature (Temp_shaft), and pad temperature (Temp_pad) become the thermal boundary conditions of the energy equation.

(2.6) Steady state of the synchronous orbit is determined by means of the Poincare plot. In this research, rotor-dynamic behavior is limited to a synchronous (1X) vibration, and fast Fourier transform (FFT) is performed at each journal orbiting to check the synchronous behavior. Figure 9 shows a schematic diagram to show the synchronous orbit over a period. If the start point and the end point meet within a predefined circle-shaped area, which is very small compared with the journal orbit size, it can be considered as a steady state; however, even if the two points meet within the predefined error criterion area, the velocity vectors defined by v0 and ve shown in Fig. 9 may not meet within the prescribed area. This cannot be considered a steady state so both the position and velocity vectors should be taken into account when defining the steady state of the synchronous journal orbit. This process is repeated until the converged orbit is produced. After the steady state of the rotor dynamic system is achieved, thermal transient analysis of the bearing and shaft is conducted.

(3.1) Heat flux boundary condition between the pad and lubricant is calculated based on: (1) the temperature distribution in the pad (TempPad), (2) orbit time averaged film thickness, and (3) the orbit time averaged temperature of the lubricant (see Sec. 2.4).

(3.2) The newly updated heat flux boundary condition works as the thermal boundary condition of the pads thermal transient analysis. The numerical integration of the thermal system is conducted from t0 to tE_therm. The final value of the thermal transient analysis, pad thermal gradient at time tE_therm, works as (1) the thermal induced film thickness change and (2) thermal boundary condition of the 3D energy equation.

(3.3) Final value of the thermal transient analysis is used for the calculation of the pad thermal deformation. Thermally deformed pad causes change of the film thickness (hpad), and the effect of the film thickness change is considered in the calculation of the fluidic force.

(4.1) Asymmetric heat flux into the synchronously whirling rotor is solved by the orbit time averaged heat flux [6] flowing from fluid film to the spinning shaft surface. The information of the fluid film temperature (TempFilm), and the film thickness (h) were saved during the THD transient analysis (see Sec. 2.4).

(4.2) Based on the newly updated heat flux boundary condition, thermal load [F] acting on the shaft thermal transient analysis is updated, and the thermal solution is gained by the numerical integration from t0 to tE_therm. The final value of the thermal transient analysis, shaft thermal gradient at tE_therm, works as (1) the thermal expansion force applied to the elastic shaft model and (2) thermal boundary condition of the 3D energy equation.

(4.3) The final value of the shaft’s thermal transient analysis is transformed into the thermal expansion force, which works in the 3D elastic shaft’s FE model. Thermal expansion of the shaft calculated by temperature distribution causes rotor thermal bending and shaft thermal expansion at the bearing location. The shaft thermal bow creates a new thermal bending
induced imbalance force ($F_{TB}$) working in the THD analysis, and thermal expansion causes a decrease in film thickness ($h_{TEJ}$).

(5) The whole process above is conducted until $t_{GLOBAL} \geq t_f$.

### 4 Rotor–Bearing Model

Morton effect predictions are presented for one rotor–bearing system presented by De Jongh and van der Hoeven’s published literature [22], and the input parameters are provided by Balbahadur and Kirk’s [9] research. Figure 10 shows the rotor dynamic model (total mass: 265 kg, total length: 1.195 m), where the linear bearing is located at node number 4 (black), and the nonlinear bearing is at 12 (gray). The linear bearing dynamic coefficients are assumed to be constant at $K_{xx} = K_{yy} = 1.70 \times 10^8$ N/m, and $C_{xx} = C_{yy} = 1.00 \times 10^5$ Ns/m, and cross coupled terms are zero.

Figure 11 shows the thermal shaft and bearing model, where seven thermal boundary surfaces are defined by red arrows. All surfaces have convective boundary conditions with ambient temperature of 30 or 40 °C as shown in Tables 1 and 2. The total length of the thermal shaft is seven times the bearing length as described in Sec. 1.3. The nondriven end (NDE) bearing input parameter is given in Table 4. The thermal bow model is considered only at the NDE bearing location. In De Jongh and van der Hoeven’s [22] test bed, the film clearance was reduced to 84.21%
of its initial value to reproduce the unstable rotor behavior at the site. In this research, the initial imbalance distribution is located on the right hand side of the NDE bearing, and three cases of nodal imbalance values are considered as shown in Table 3. The Morton effect problems in the test bed described in De Jongh and van der Hoeven’s research [22] are as below:

(a) Above 8000 rpm, the rotor started to show unstable behavior at any spin speed.
(b) Within 1 or 2 min it reached vibration trip level.
(c) Cyclic oscillations in the phase of amplitude could be seen.
(d) Hysteresis could be seen in the vibration amplitudes differential between run-up and coast-down.

5 Three-Dimensional Energy Equation

Gomiciaga and Keogh [6] adopted a 3D energy equation approach to predict the thermal gradient inside the synchronously orbiting shaft, and their research was limited to a prescribed journal orbit without a dynamic model. Childs and Saha [12] and Lee and Palazzolo’s [13] studies considered a 2D energy equation to predict the thermal bow induced rotor instability problem. In this section static equilibrium analysis with zero size orbit and transient Morton effect simulation with large size orbit will be performed using both 2D and 3D energy equations, and the results will be examined.

### Table 1 Shaft’s thermal boundary condition

<table>
<thead>
<tr>
<th>Ambient temperature (°C)</th>
<th>Convection coefficient (W/m² K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left end</td>
<td>30</td>
</tr>
<tr>
<td>Left side</td>
<td>30</td>
</tr>
<tr>
<td>Right side</td>
<td>30</td>
</tr>
<tr>
<td>Right end</td>
<td>30</td>
</tr>
</tbody>
</table>

| Case 1 | 10.00 x 10⁻⁵ | Case 2 | 3.00 x 10⁻⁵ | Case 3 | 1.00 x 10⁻³ |

### Table 2 Pads’ thermal boundary condition

<table>
<thead>
<tr>
<th>Ambient temperature (°C)</th>
<th>Convection coefficient (W/m² K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing radial out</td>
<td>40</td>
</tr>
<tr>
<td>Axial left end</td>
<td>30</td>
</tr>
<tr>
<td>Axial right end</td>
<td>30</td>
</tr>
</tbody>
</table>

### Table 3 Simulation model parameters

<table>
<thead>
<tr>
<th>Lubricant parameters</th>
<th>Bearing/pad parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity at 50°C (N s/m²)</td>
<td>0.0203</td>
</tr>
<tr>
<td>Viscosity coefficient (1/°C)</td>
<td>0.031</td>
</tr>
<tr>
<td>Supply temperature (°C)</td>
<td>50</td>
</tr>
<tr>
<td>Inlet pressure (Pa)</td>
<td>1.32 x 10⁵</td>
</tr>
<tr>
<td>Reference temperature (°C)</td>
<td>50</td>
</tr>
<tr>
<td>Mixing coefficient (β)</td>
<td>0.8</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>860</td>
</tr>
<tr>
<td>Heat capacity (kJ/kg °C)</td>
<td>2000</td>
</tr>
<tr>
<td>Heat conductivity (W/m K)</td>
<td>0.13</td>
</tr>
<tr>
<td>Shaft/rotor parameters</td>
<td>Number of pads 5</td>
</tr>
<tr>
<td>Young’s modulus (Pa)</td>
<td>2.10 x 10¹¹</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>7850</td>
</tr>
<tr>
<td>Heat capacity (kJ/kg °C)</td>
<td>453.6</td>
</tr>
<tr>
<td>Heat conductivity (W/m K)</td>
<td>50</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Thermal expansion coefficients (1/°C)</td>
<td>1.30 x 10⁻⁵</td>
</tr>
<tr>
<td>Reference temperature (°C)</td>
<td>25</td>
</tr>
<tr>
<td>Thermal shaft length (m)</td>
<td>0.3556</td>
</tr>
<tr>
<td>Pad type</td>
<td>Number of pads 5</td>
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<tr>
<td>Pad arc length (deg)</td>
<td>56</td>
</tr>
<tr>
<td>Offset</td>
<td>0.5</td>
</tr>
<tr>
<td>Radius of shaft (m)</td>
<td>0.0508</td>
</tr>
<tr>
<td>Bearing clearance (m)</td>
<td>7.49 x 10⁻⁵</td>
</tr>
<tr>
<td>Preload</td>
<td>0.5</td>
</tr>
<tr>
<td>Pad thickness (m)</td>
<td>0.0127</td>
</tr>
<tr>
<td>Bearing length (m)</td>
<td>0.0508</td>
</tr>
<tr>
<td>Young’s modulus (Pa)</td>
<td>2.1 x 10⁵</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Thermal expansion coefficients (1/°C)</td>
<td>1.30 x 10⁻⁵</td>
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<td>Density (kg/m³)</td>
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</tr>
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<td>Heat capacity (kJ/kg °C)</td>
<td>453.6</td>
</tr>
<tr>
<td>Heat conductivity (W/m K)</td>
<td>50</td>
</tr>
<tr>
<td>Reference temperature (°C)</td>
<td>25</td>
</tr>
<tr>
<td>Linear bearing (node 4)</td>
<td>Kxx, Kyy (N/m) 1.70 x 10⁸</td>
</tr>
</tbody>
</table>

### Table 4 Initial imbalance distribution

<table>
<thead>
<tr>
<th>Node</th>
<th>Imbalance (kgm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>14,15,16,17,18,19</td>
</tr>
<tr>
<td>Case 2</td>
<td>14,15,16,17,18,19</td>
</tr>
<tr>
<td>Case 3</td>
<td>14,15,16,17,18,19</td>
</tr>
</tbody>
</table>

5.1 Static Equilibrium Analysis. The static equilibrium analysis of the rotor–bearing system with the input parameters shown in Table 1 was conducted using both 2D and 3D energy equations.

Figure 12 compares the dynamic coefficients, journal static condition, and temperatures calculated both by the 2D and 3D energy equations over the spin speed range from 4500 to 15,500 rpm. Both heat conduction and thermal deformation are taken into account in the shaft and bearing model using the 3D FE model.

In the case of the dynamic coefficients, static eccentricity and mean fluid film temperature, both 2D and 3D energy equations produce very similar results, and the differences are negligible as can be in Figs. 12(a)–12(f). The journal attitude angle differential between the 2D and 3D energy equations increases with the spin speed yielding 2 deg at 15,500 rpm as seen in Fig. 12(e). The peak and average lubricant temperatures are compared in Figs. 12(c) and 12(f) that produce a linear trend between the rotor spin speed and temperature. The peak temperature differential is 5°C at the highest spin speed as seen in Fig. 12(c). The root cause of the higher temperature of the 3D energy equation is thought to be the viscous shearing arising from the axial fluid flow considered only in the 3D energy equation.

In practice, the 2D energy equation is solved more than 50 times faster than the 3D energy equation due to its smaller FE matrix size. Hence, the 2D energy equation is a good assumption and suitable for the static equilibrium analysis for its fast calculation ability.

5.2 Transient Morton Effect Analysis. In Sec. 5.1, the 2D energy equation was proved to be a good assumption for the static equilibrium analysis, when the orbit size is zero. This section examines the transient rotor-dynamic analysis with a large orbit size using both 2D and 3D energy equations, where the journal differential heating and the resultant thermal imbalance force are taken into account. The rotor–bearing input parameters are identical to those in the previous section. In the case of the initial
Fig. 12  Steady state analysis of rotor–bearing system with 2D and 3D energy equations

Fig. 13  Comparison of 2D and 3D energy equation for transient Morton effect analysis
imbalance, case 1 in Table 3 is selected to produce a bigger orbit and a resultant higher viscous shearing. Rotor spin speed remains constant at 8500 rpm.

Figure 13(a) shows the variation of journal surface temperature differential with the axial position at $t = 3$ (min). $L$ indicates the bearing length, and $z$ is the axial position so $|z/L| = 0.5$ on the $x$ axis indicates both ends of the bearing. The journal temperature differential of the 2D energy equation shows four times as large as the result of the 3D energy equation. It can be seen that the 2D energy equation overpredicts the temperature differential all over the thermal shaft length.

In Sec. 1.3, seven times the bearing length was used as the thermal shaft length and the journal temperature differential at the end of the shaft decays to zero, as can be seen in Fig. 13(a). The thermal shaft length should be long enough for the surface temperature differential to converge to zero at both ends of the thermal shaft for the exact thermal bow calculation. If the temperature differential is not zero at both ends, the length should be increased since the short length may produce an underestimated thermal bending angle leading to an underestimated Morton effect simulation result.

The vibration amplitudes at the NDE bearing and that at the end of the overhung disk are shown in Figs. 13(b) and 13(c), respectively. The vibration amplitude differential between the 2D and 3D energy equations at the end of the disk is 3.5 times as high as at the NDE bearing location. So it can be seen that the higher temperature differential of the 2D energy equation caused the bigger thermal bow angle leading to the increased vibration level at the end of the rotor.

The 1X filtered polar plot is drawn in Fig. 13(d). In both cases, the moving phase shows a continuous change of unbalance vector as in the phase of the moving hot spot seen in Fig. 13(e). One of the important factors that can make a significant change in the Morton effect problem is the phase information. The moving phase of the hot spot is plotted in Fig. 13(e) where the change rate of the 2D energy equation is higher than the 3D energy equation. A bigger thermal induced imbalance vector of the 2D energy equation approach can be seen as the cause of the higher change rate of the hot spot phase.

The bigger thermal induced imbalance is found to be the root cause of the higher vibration amplitude in this section. This process decreases the minimum film thickness, and finally causes bearing failure due to the rubbing problem as can be seen in Fig. 13(f), where the temperature differential becomes divergent by approaching the bearing clearance.

It is evident that the 2D energy equation is neither reliable nor suitable for the Morton effect simulation with large orbit due to its: (1) overpredicted vibration amplitude, (2) overpredicted journal surface temperature differential, and (3) different change rate of the hot spot phase.

6 Simulation Results and Discussion

6.1 Effect of Thermal Bow. The asymmetric temperature across the journal diameter caused the rotor to bow at the NDE bearing location, inducing an imbalance of overhung mass,
changing the vibration amplitude and phase of synchronous vibrations, and the journal's asymmetric temperature. In this section the thermal bow induced synchronous whirl phenomenon is studied by comparing nonthermal bow and thermal bow models at 8500 rpm constant spin speed. The rotor–bearing system and the thermal boundary conditions are identical to the earlier described rotor–bearing model shown in Tables 1–4, and the initial imbalance distribution is case 2 of Table 4. The rotor–bearing configuration of the nonthermal bow model is identical to the thermal bow model except for the thermal bow induced imbalance distribution; in other words, the only imbalance force acting on the nonthermal bow model is the initial mechanical imbalance.

Figure 14(a) shows the journal surface temperature differential at the bearing center in the axial direction. The journal temperature differential of the thermal bow model oscillates and increases with time. However, the temperature differential of the nonthermal bow model does not oscillate but decays to 1 °C. Figure 14(b) shows the vibration amplitude at the NDE bearing. The vibration level in common with the p–p temperature oscillates over the period of 3 min. On the other hand, the nonthermal bow model does not show any oscillation or an increase of vibration level. In Fig. 14(c), the thermal bow model shows a moving phase lag relative to the initial imbalance vector and the nonthermal bow model shows a steady phase lag. It is evident that the thermal bow effect makes a significant change in the rotor-dynamic behavior; this will be further investigated in part II of this research.

6.2 Hysteresis Vibration Amplitude Curve With Varying Spin Speed. De Jongh and Morton [23] and De Jongh and van der Hoeven’s [22] literatures showed the hysteresis vibration with time varying spin speed by experimental studies. The hysteresis vibration amplitude is one of the easily recognized phenomena of the Morton effect problem. In this section, the hysteresis phenomenon is simulated with time varying spin speed as shown in Fig. 15(a). The rotor–bearing system is shown in Tables 1–4 and the initial imbalance is case 2 of Table 4. Rotor spin speed was gradually increased from 5000 to 7500 rpm with a growth rate of 893 (rpm/min) as can be seen in Fig. 15(a). At 7500 rpm, the vibration level was 1 × 10⁻⁶ m. When the rotor was accelerated up to 8500 rpm and the spin speed stayed at around 8500 rpm for 36 s, the vibration amplitude was increased to 3.5 × 10⁻⁶ m that is 350% higher than 7500 rpm. Figure 15(b) shows a bode plot, where the hysteresis vibration with the time varying speeds can be observed. Figure 15(c) describes the thermal bow vector versus initial imbalance vector. Since the hot spot tends to move around the journal surface counterclockwise, that is the same as the shaft spinning direction, the thermal induced imbalance vector cannot go back the way it came when the spin speed is reduced. The residual thermal bow and the moving thermal bow are the main causes of the hysteresis vibration.

7 Conclusions

(a) The earlier studies [9–13] adopted analytical methods for the evaluation of the thermal bow angle and phase, assuming that the thermal bow phase coincides with the cold spot. On the other hand, the present work presented a new approach for the thermal bow eccentricity and phase, where the thermal gradient and the heat conduction inside the 3D thermoelastic FE shaft model are employed.

(b) 3D film thickness formula is employed to simulate the 3D rotor–bearing dynamic motion.

(c) Asymmetric shaft thermal expansion model is introduced, where the temporal derivative of the film thickness due to the spinning shaft is taken into account.

(d) Modified mixing temperature theory is presented, where the recirculated flow from the upstream pad to the downstream pad is limited by adopting the mixing coefficient

factor. For the 3D bearing model, the axial inlet temperature variation is considered.

(e) The 2D energy equation is found to be a good assumption, and suitable for the static equilibrium analysis, where the journal differential heating can be ignored and the orbit size is zero. However, the Morton effect simulation with large orbit showed different results. The vibration amplitude at the NDE bearing location of the 2D energy equation was 350% higher than the 3D energy equation, and the journal surface temperature differential of the 2D energy equation was 400% higher than the 3D energy equation. The 3D energy equation is found to be suitable for the Morton effect simulation.

(f) The thermal bow effect makes a significant change to the dynamic behavior of the rotor bearing system. Hysteresis vibration amplitude with the time varying rotor spin speed was observed, which is the noticeable characteristic of the synchronous thermal instability problem.

Acknowledgment
The authors gratefully acknowledge support of this research from the Texas A&M Turbomachinery Research Consortium (TRC) member companies.

Nomenclature

- \( c \): heat capacity
- \( C_{Brg} \): bearing damping matrix
- \( C_{gyro} \): gyroscopic matrix
- \( C_{L} \): bearing clearance
- \( C_{Lp} \): pad clearance
- \( d_n \): distance between \( n \)th node and thermal shaft right end
- \( e_G \): distance between mass center and pivot
- \( e_n \): thermal bow induced eccentricity at \( n \)th node of FE rotor
- \( e_x \): \( x \) component of journal position
- \( e_y \): \( y \) component of journal position
- \( F_{b} \): fluidic force acting on pad
- \( F_{R} \): external force acting on FE rotor
- \( F_{thb} \): thermal bow induced imbalance force acting on \( x \) axis
- \( F_{yfb} \): thermal bow induced imbalance force acting on \( y \) axis
- \( h \): convective coefficient
- \( h_{Tef} \): film clearance reduction due to journal thermal expansion
- \( h_{tep} \): film clearance reduction due to pad thermal expansion
- \( J_G \): moment of inertia of pad
- \( k \): heat conductivity
- \( K_{Brg} \): bearing stiffness coefficient matrix
- \( K_{struc} \): FE rotor stiffness matrix
- \( l_e \): axial position of shaft right end
- \( m_B \): pad mass
- \( M_n \): mass at \( n \)th node of FE rotor
- \( M_p \): fluidic moment acting on pivot
- \( M_R \): mass matrix of FE rotor model
- \( N_{Rotor} \): number of node of FE rotor model
- \( p \): fluid pressure
- \( P \): modal thermal load
- \( p_{pivot} \): pivot displacement

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References


