The Fault-Tolerant Control of Magnetic Bearings With Reduced Controller Outputs

1 Introduction

Magnetic bearings are filling greater applications in industry since they have many advantages over conventional fluid film bearings such as elimination of lubrication, less power loss, and operation at temperature extremes. Furthermore, active stiffness and damping properties of magnetic bearings make it possible to adjust rotordynamic properties while operating the rotor-bearing system. Synchronous vibration due to imbalance can also be successfully reduced by automatic balancing [1]. The modeling and design of magnetic bearings have been investigated by many researchers [2–4]. However, reliability requirements limit magnetic bearings from being used in many potential applications. Fault tolerant control of heteropolar magnetic bearings has been investigated by several researchers to improve reliability.

Maslen and Meeker [5] developed a fault-tolerant control scheme of a 8-pole heteropolar magnetic bearing with independently controlled currents. Their approach utilizes the flux coupling property of heteropolar magnetic bearings. Redefining remaining coil currents with the distribution matrix, when one or more coils are disconnected, makes it possible to produce desired force resultants with the help of flux coupling. However, flux coupling with independent currents results in an electromagnet stability problem. Current states become unstable because one of the eigenvalues of the rank-deficient inductance matrix is zero, which results in power amplifier limiting [6]. Meeker [6] introduced the decoupling choke to remedy this stability problem. In essence, to use a decoupling choke means that all coils from the magnetic bearing are wound around a common, external electromagnet core, then return to their power amplifiers. The number of coil turns in the decoupling choke can be designed to produce additional inductance so that the inductance matrix should be completely nonsingular. The fault-tolerant magnetic bearings were demonstrated on a large flexible rotor [7].

This fault-tolerant control scheme has an advantage over the 3 control-axis redundant control scheme developed by Lyons [8] with regard to fault tolerance of multiple failed poles. The 3 control-axis redundant control scheme will not work if 2 of the 3 control axes experience any number of coil failures. This advantage of the former over the latter approach does require an increase in hardware complexity for implementation. The fault-tolerant control scheme of an 8-pole heteropolar magnetic bearing with independently controlled currents requires 8 digital controller output channels while the 3 control-axis redundant control scheme requires only 3 digital controller output channels. Therefore, 16 output channels are required for the control of 2 redundant radial bearings. Furthermore, a decoupling choke is required for each bearing. This requirement for extensive hardware may prevent the control scheme from being used in some industrial applications.

Reducing the number of digital controller outputs and eliminating decoupling chokes makes the fault tolerant control scheme more practical for industrial applications. Fault tolerant schemes that only require 3 controller outputs for a radial bearing may be implemented on a commercial 8-channel DSP controller for fault tolerant control of a five-axis rotor-bearing system.

2 Fault Tolerance of Grouped Magnetic Bearings

Fluxes can be isolated for a multiple-pole magnetic bearing when coils are wound in series with a second pole of opposite polarity. Figure 1 shows that control currents of the 12-pole heteropolar magnetic bearing can be separated into 3 groups (6 coil pairs, where a coil pair is two coils wound in series on different poles). The flux vector of the 1-D magnetic circuit is [5]

\[ \phi(x, y) = R(x, y)^{-1}NI = V(x, y)I \]  

(1)

where \( R, N, \) and \( I \) represent the reluctance matrix, coil turn matrix, and the current vector, respectively. The current vector with coil coupling is

\[ I = N_I \hat{I} \]  

(2)

The flux vector evaluated at the bearing center position is

\[ \phi_{x=0} = \frac{n a \mu_0}{g_0} N_I \hat{I} \]  

(3)

where \( a, \mu_0, g_0, \) and \( n \) represent pole face area, permeability of air, nominal air gap, and number of coil turns, respectively. Equation (3) shows that all fluxes are isolated in C-cores (equal magnitude and opposite polarity type paths). Note that the 6 C-core
The electric circuit equation for the power amplifiers loads is

\[ L(x, y) \frac{\partial \vec{I}}{\partial t} + \Re \vec{I} = V_s \]  

(4)

where the inductance matrix evaluated at the bearing center position is given as

\[ L_{10} = \frac{n^2 a \mu_0}{g_0} N_x^3 N_y \]

\[ L_{10} = \begin{bmatrix}
-1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0
\end{bmatrix} \]

(5)

The matrix \( H \) is also used to define the grouping of currents with the multiple coils failed bearing. For example, \( H \) for 2-3-6-7th poles (2-4th coils) failed bearing is given as

\[ H = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \]

(6)

The control current output vector \( I_0 \) is defined as

\[ I_0 = \kappa TV_c \]

(7)

where \( V_c = [v_{b}, v_{c-x}, v_{c-y}]^T \) and where \( \kappa \) represents the power amplifier’s DC gain. \( v_b, v_{c-x}, \) and \( v_{c-y} \) are bias, \( x \) control, and \( y \) control voltage outputs, respectively. The distribution matrix \( T \) is defined as

\[ T = [T_b, T_x, T_y] \]

(8)

where

\[ T_b = [t_1, t_2, t_3]^T, \quad T_x = [t_4, t_5, t_6]^T, \quad T_y = [t_7, t_8, t_9]^T \]

(9)

The current distribution matrix \( T \) must be determined so that magnetic forces in the grouped bearing are linearized and decoupled even in case of multiple coils failed. The necessary conditions for force linearization and uncoupling are [5,13]

\[ T^T G = M \]

(10)

where

\[ G = -H^T V_c \frac{\partial D}{\partial \phi} VH \]

\[ M_1 = \begin{bmatrix}
0 & 0.5 & 0 \\
0 & 0 & 0 \\
0.5 & 0 & 0
\end{bmatrix}, \quad M_1 = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0.5 & 0 & 0
\end{bmatrix} \]

To yield the linearized forces:

\[ F = v_b v_{c-y} \]

(11)

If there exists a distribution matrix \( T \) that satisfies Eq. (12) for a specific failure case, a set of 3 independent currents will be distributed into 3 groups of poles to produce desired force resultants in the \( x \) and \( y \) directions.
3 Determination of Distribution Matrices

The distribution matrices for the fault-tolerant bearing with reduced control outputs are calculated using a Lagrange Multiplier optimization method. The cost function of the weighted norm of the flux density vector is

$$J(T) = B(T)^TPB(T) = T^TV^TPV$$

Subject to the equality constraints (12)

$$T^TG_xT - M_x = 0$$

Although a diagonal weighting matrix $P$ can be assigned to increase load capacity in a specific direction, an identity matrix is selected here for simplicity. Twelve equality constraint equations are derived from Eqs. (15) with $\varphi \in \{x,y\}$

$$h_1(T) = T^G_yG_xT_b$$
$$h_2(T) = T^G_yG_xT_i - 0.5 = 0$$
$$h_3(T) = T^G_yG_xT_f = 0$$
$$h_4(T) = T^G_yG_xT_s = 0$$
$$h_5(T) = T^G_yG_xT_v = 0$$
$$h_6(T) = T^G_yG_xT_w = 0$$

(16)

$$h_7(T) = T^G_yG_xT_b$$
$$h_8(T) = T^G_yG_xT_b$$
$$h_9(T) = T^G_yG_xT_b$$
$$h_{10}(T) = T^G_yG_xT_b$$
$$h_{11}(T) = T^G_yG_xT_b$$
$$h_{12}(T) = T^G_yG_xT_b$$

The Lagrange Multiplier method is applied to the problem of solving for the $T$ that satisfies Eq. (12). Define:

$$\hat{L}(T) = J(T) + \sum_{i=1}^{12} \lambda_i h_i(T)$$

where $\lambda_i$ are Lagrange multipliers. Partial differentiation of Eq. (17) with respect to $t_i$ and $\lambda_i$ leads to 21 nonlinear algebraic equations to solve for $t_i$ and $\lambda_i$.

$$w_j = \frac{\partial L}{\partial t_i} = 0, \quad i = 1, 2, \ldots, 9$$

The vector form of these equations is

$$\begin{bmatrix}
    w_1(t,\lambda) \\
    w_2(t,\lambda) \\
    \vdots \\
    w_{12}(t,\lambda)
\end{bmatrix} = \begin{bmatrix}
    0 \\
    0 \\
    \vdots \\
    0
\end{bmatrix}$$

$$W(t,\lambda) = \begin{bmatrix}
    w_1(t,\lambda) \\
    w_2(t,\lambda) \\
    \vdots \\
    w_{12}(t,\lambda)
\end{bmatrix} = \begin{bmatrix}
    0 \\
    0 \\
    \vdots \\
    0
\end{bmatrix}$$

(20)

The distribution matrices $T$ are calculated by solving the system of nonlinear algebraic equations shown in Eq. (20). A nonlinear algebraic equation solver using a least square iterative method (MATLAB) was used to solve Eq. (20) numerically. Various initial guesses of $t_i$ and $\lambda_i$ are used in order to find the converged solutions. The 12-pole heteropolar magnetic bearing used in this analysis has uniform pole face area $a$ of $1.6058 \times 10^{-2}$ m$^2$, a nominal gap $g_0$ of $5.08 \times 10^{-4}$ m, and number of coil turns $n$ of 100. The distribution matrix $T$ is always a $3 \times 3$ matrix for any failure case in this “3 group” strategy.

Several distribution matrices were calculated for the unfailed and failed bearings. A valid $T$ matrix for a 12-pole/3 group heteropolar magnetic bearing in unfailed coil operation is given as

$$T_u = \begin{bmatrix}
    0.45108 & 0.0488 & 0.03385 \\
    0.45108 & -0.049 & 0.033825 \\
    -0.45108 & 0.000075 & 0.050875
\end{bmatrix}$$

(21)

A distribution matrix for the 2 coil pairs failed bearing (2–4th coil pairs failed) is

$$T_2 = \begin{bmatrix}
    0.36694 & 0.10825 & 0.041664 \\
    0.51893 & -0.10205 & 0.058921 \\
    0.36694 & -0.072163 & -0.0662496
\end{bmatrix}$$

(22)

A distribution matrix for the 3 coil pairs failed bearing (2–4–6th coil pairs failed) is

$$T_3 = \begin{bmatrix}
    0.93579 & 0.054469 & 0.081712 \\
    0.93579 & -0.054421 & 0.081712 \\
    -0.4679 & 0 & 0.10061
\end{bmatrix}$$

(23)

4 Control System Design

The calculated distribution matrices, which work like adaptive gain matrices to compensate for missing coils, can be implemented in a physical controller, so if some combinations of failures of the power amplifiers or coils are detected, the corresponding distribution matrices can be switched shortly thereafter. Figure 2 shows a diagram for illustrating the operation of the fault-tolerant controller. The inputs are the 2 position signals and the reference bias voltage $v_b$. Both control gains and distribution matrices must be adjusted for the specific configuration of coil failures. This fault-tolerant scheme only requires 3 DSP controller outputs for a radial bearing, so a commercial 8-channel DSP controller can be used for the fault tolerant control of a five-axis rotor-bearing system.

The nonlinear magnetic force in Eq. (6) can be linearized about the bearing center position and the zero control voltages by using Taylor series expansion [13,14]. The bias voltage gain can be adjusted for the given distribution matrix so that the maximum component of the bias flux density vector is equal to $b_{sat}/2$, to generate maximum load. The parameter $b_{sat}$ represents the saturation flux density. The linearized forces are

$$F_i = -K_{pxx}x - K_{pvy}y + K_{exx}v_{ex}$$

(24)

$$F_y = -K_{pxy}x - K_{pyy}y + K_{eyy}v_{ey}$$

(25)
The position stiffnesses are described as

\[ K_{p \varphi \varphi} = -\sigma^2 k T^2 H U_{\varphi \varphi} H T_{\varphi} \dot{\varphi}_0 \]  

and voltage stiffnesses are described as

\[ K_{v \varphi \varphi} = 2\sigma^2 k T^2 H U_{\varphi \varphi} H T_{\varphi} \dot{\varphi}_0 \]  

where

\[ U_{\varphi \varphi} = -V \frac{\partial D}{\partial \varphi} V^T \bigg|_{\varphi = 0}, \quad U_{\dot{\varphi} \varphi} = -2V \frac{\partial D}{\partial \dot{\varphi}} \frac{\partial V}{\partial \varphi} \bigg|_{\varphi = 0} \]

and where both \( \varphi \) and \( \omega \) represent either \( x \) or \( y \). The position stiffnesses and voltage stiffnesses are calculated for the distribution matrices of \( T_x \) and \( T_y \) for the given bias voltages as shown in Table 1.

The simple PD control with low pass filters is used to design the closed-loop system for unfailed bearings. The closed-loop dynamic properties can be calculated when the control gains are appropriately selected [15]. The same closed-loop stiffnesses and dampings may be maintained before and after coil failure if control gains are switched to appropriate values. The decoupled linearized forces for an unfailed bearing are

\[ F_{q \varphi}^N = -K_{p \varphi \varphi}^N + K_{v \varphi \varphi}^N \dot{\varphi}_0 \]  

where

\[ \dot{\varphi}_0 = \xi \varphi \]  

and where \( \xi \) is the sensor sensitivity. In general, the linearized forces for the failed bearing have undesirable cross-coupled position stiffnesses, and the direct position stiffnesses along the \( x \) and \( y \) axes are usually not symmetric. Cross feedback control forces are added in the linearized force equations of the failed bearing in order to cancel out the cross coupled position stiffnesses. The linearized forces for the failed bearing are

\[ F_x^F = -K_{p xx}^F x - K_{p xy}^F y + K_{v xx}^F (\dot{x}_c + \dot{u}_c) \]

\[ F_y^F = -K_{p xy}^F x - K_{p yy}^F y + K_{v xy}^F (\dot{y}_c + \dot{u}_c) \]

Table 1 The calculated stiffnesses

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<th>( \varphi )</th>
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<th>( T_y )</th>
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<td>( K_{v xx} ) (N/Volt)</td>
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<td>5</td>
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5 Simulations

The fault-tolerant control system is simulated for multiple coils failed cases on a horizontal rigid rotor supported with 12-pole heteropolar bearings. The schematic of the rotor-bearing system is shown in Fig. 3. The symmetric rotor used in this analysis has mass of 10 kg, polar moment of inertia of 0.05 kgm², and transverse moment of inertia of 0.2 kgm² about the mass center. Two magnetic bearings are located at 0.1 m either side of the mass center. An unbalance eccentricity of \( 40 \times 10^{-6} \) m for the total rotor mass is applied at both bearing locations with relative phase angle 90 deg. Power amplifier and sensor dynamics are included in the closed-loop system with a power amplifier gain \( \kappa \) of 1 amper/volt and sensor sensitivity \( \xi \) of 7874 volt/m. The power amplifier dynamics is \( PA(s) = 1/(\tau_p s + 1) \), where \( \tau_p = 1/(2\pi f_p) \), and \( f_p = 2500 \) Hz. The sensor dynamics is given as \( S(s) = 1/(\tau_s s + 1) \), where \( \tau_s = 1/(2\pi f_s) \), and \( f_s = 3000 \) Hz.

The transient response from normal operation with no failure to fault-tolerant control with 2–4th coil pairs failed for both bearings was simulated for nonlinear bearings at 10,000 rpm. The selected control gains of \( K_{p xx}^N, K_{p yy}^N, K_{v xx}^N, \) and \( K_{v yy}^N \) for the unfailed operation are 80, 80, 0.2, and 0.2, respectively. The required control gains of \( K_{p xx}^F, K_{p yy}^F, K_{v xx}^F, \) and \( K_{v yy}^F \) to maintain the same closed-loop dynamic properties are 73.46, 75.95, 0.2, 0.2, -0.0263, and -0.0272, respectively.
Transient response of the orbit at bearing A is shown in Fig. 4. The orbit plot shows that the rotor sags slightly due to gravity and makes good orbits for normal operation. The distribution matrix of $T_u$ was switched to $T_2$ when 2 coil pairs failed at 0.1 second. The orbit becomes elliptic after failure. Transient response of the current inputs to bearing A for the 2–4th coil pairs failed case is shown in Fig. 5. This shows that large amount of currents to the unfailed poles are required to maintain the similar orbits before and after failure. Transient response of the flux densities to bearing A for the 2–4th coil pairs failed case is shown in Fig. 6. It is notable that the fluxes in 2–3–6–7th poles are completely eliminated with the distribution matrix of $T_2$.

6 Conclusions

With the proposed fault-tolerant control scheme the number of required controller outputs is greatly reduced. Since this fault-tolerant scheme only requires 3 controller outputs per radial bearing, a commercial 8-channel DSP controller is sufficient to implement five-axis redundant control. Unlike the requirements of the Maslen and Meeker approach, decoupling chokes are not required for the proposed redundant control with 6 power amplifiers. The control scheme utilizes flux isolation (C core) property of heteropolar magnetic bearings. Compared to the previous approach [5] with an 8-pole bearing, this control scheme reduces controller outputs by half or more, and removes decoupling choke requirement. This will facilitate fault-tolerant control of heteropolar magnetic bearings for industrial applications. Simulations of a horizontal rigid rotor supported on two 12 pole bearings with grouped currents show that the system maintains good control even when multiple coils fail simultaneously.

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References


