Magnetically Suspended VSCMGs for Simultaneous Attitude Control and Power Transfer IPAC Service

This paper presents the theory and numerical results of utilizing four gimbaled, magnetically suspended, variable speed flywheels for simultaneous satellite attitude control and power transfer (charge, storage, and delivery). Previous variable speed control moment gyro models and control algorithms assumed that the flywheel bearings were rigid. However, high speed flywheels on spacecraft will be supported by active magnetic bearings, which have flexibility and in general frequency dependent characteristics. The present work provides the theory for modeling the satellite and flywheel systems including controllers for stable magnetic bearing suspension for power transfer to and from the flywheels and for attitude control of the satellite. A major reason for utilizing flexible bearings is to isolate the imbalance disturbance forces from the flywheel to the satellite. This g-jitter vibration could interfere with the operation of sensitive onboard instrumentation. A special control approach is employed for the magnetic bearings to reject the imbalance disturbances. The stability, robustness, tracking, and disturbance rejection performances of the feedback control laws are demonstrated with a satellite simulation that includes initial attitude error, system modeling error, and flywheel imbalance disturbance. [DOI: 10.1115/1.4002105]

1 Introduction

Recent work seeks to demonstrate a reduction in satellite weight and cost by replacing the present energy storage system (electrochemical batteries) and attitude control torque actuators (flywheel and/or control moment of gyro) with an array of four high speed flywheels (FW) or variable speed control moment gyro (VSCMGs), which have merits of both flywheels and control moment gyro (CMG). Utilizing high speed flywheels for energy storage on the satellites was suggested by Roes [1] as early as 1961 and the advantages of magnetic bearing (MB) suspension of flywheels for attitude control and energy storage were discussed by Sindlinger [2] and Brunet [3]. NASA related flywheel R&D contains the pioneering work of Kirk et al. [4–7] for improving energy density and for incorporating magnetic bearings.

The effects of incorporating a magnetic bearing on satellite nutation instability is analyzed in Heimbold [8]. This paper shows that damping between rotating and fixed components may cause an overall nutational instability. This was based on an approximate (linearized) analysis and an interpretation of prior research by Mingori [9,10] and Gunter [11]. Mingori’s work focused on the effects of damping in the interconnection between the rotating and “fixed” parts of a dual-spin satellite, and Gunter’s work focused on fluid film bearings in high speed turbomachinery. Although Gunter showed that damping within the rotating member is destabilizing his work, it also demonstrated that bearing damping that is damping between the rotating (shaft) and fixed (housing) components, is stabilizing. Therefore, it is our opinion that Heimbold’s interpretation of Gunter’s work as a support of his hypothesis was incorrect. We do support Heimbold’s argument that eddy currents in magnetic bearings are destabilizing due to loss in phase margin in the feedback path. This can be compensated for by direct damping in the bearing or by cross axes coupling as stated by Heimbold, “the nutational instability can be avoided by interaxes cross coupling of the bearing tilt controllers.” A detailed discussion of cross axes coupling for flywheel magnetic bearing control was provided in Park [12]. Tonkin [13] postulated that large deflections of the flywheel in its bearings due to bearing compliance may cause a satellite, open loop, nutation mode instability, if nutation occurs in the same sense as the flywheel spin direction. This phenomenon was not detected in the simulations of the present work, perhaps because of the magnetic bearing control feedback loop keeping the deflections of the shaft relative to the flywheel housing at very small amplitudes.

The work of Kenny et al. [14] integrated sensorless field oriented motor control, which was successfully demonstrated at 60,000 rpm on a NASA flywheel. Christopher and Beach provided a comprehensive overview of the NASA Glenn flywheel program in Ref. [15]. Successful integrated power and attitude control (IPAC) demands a control torque approach that uncouples the attitude control torque and power transfer torque to isolate each function during satellite maneuvers or power transfers. This is accomplished by obtaining the attitude control torque from the range space and the power transfer torque from the null space of the rectangular input-torques related matrix [16,17].

Tsiotras, et al. [18] published a remarkable series of papers related to integrated power and attitude control for a satellite with an array of high speed flywheels. Tsiotras [18] developed an algorithm for controlling the spacecraft attitude while simultaneously tracking a desired power profile by using a cluster of more than three noncoplanar energy/momentum wheels. The torque was decomposed into two perpendicular spaces; one to track the required power level of the wheels and one to control the attitude of the satellite. A logarithm term was introduced for a kinematical parameter in the Lyapunov function making the feedback controller corresponding to this parameter become linear. The work of Yoon and Tsiotras [19] extended their earlier IPAC control developments to include actuation and energy storage with single-gimbaled, variable speed control moment gyroscopes. Yoon and Tsiotras [20] designed an adaptive control law for spacecraft attitude tracking utilizing VSCMGs with uncertainties in the gimbal axes directions. This effect may be very important for IPAC.
applications since the flywheels have high speeds and high angular momentum. Their results demonstrate robust control. Yoon and Tsiotras [21] introduced a gradient based method using null motion to avoid the singularities of a VSCMG cluster. Geometric and algebraic considerations are provided to determine whether the VSCMGs will encounter an inescapable singularity when operated in integrated power and attitude control service. Jung and Tsiotras [22] demonstrated successful implementation of their integrated power and attitude control developments on an experimental test rig. Their platform is based on a realistic 3-dof spacecraft simulator equipped with four VSCMGs. The results validated their theoretical work in the areas of an adaptive control law, a power tracking control law, and wheel speed equalization.

Schaub et al. [23] published a nonlinear feedforward/feedback controller for a prototype for large three dimensional rotational craft simulator equipped with four VSCMGs. The results demonstrated successful implementation of their instinctive suspended, VSCMG module. The equations of the gimbal configuration variable speed control moment gyros are developed in Ref. [24]. VSCMGs combine the advantages of the classical single-gimbal CMG and reaction wheel (RW)/flywheel. It has the rotational speed of the RW/flywheel and the precession rate of a CMG. Two different control based steering laws are introduced from the Lyapunov stability approach and compared by numerical simulation with the classical CMG approach. The weighted pseudo inverse was utilized to obtain required torques for the satellite attitude and angular velocity error regulation problem. The simultaneous attitude control and energy storage using four standard pyramid configuration VSCMGs were presented in the Ref. [16] and Euler parameters and velocity based steering law were employed for attitude kinematics and feedback control law. The attitude control torques and power tracking torques are obtained from the range space and null space of dynamic matrix, which is not square. The prior developments for the dynamics, feedback control law, and integrated power and attitude control system (IPACS) literatures for VSCMGs treated the flywheel bearings as rigid and with perfectly mass balanced flywheels, so as to neglect the mass imbalance sinusoidal disturbance, which occurs at its spin speed. This approach simplifies the model by assuming that each flywheel motions could be effectively modeled with a single degree of freedom (allows only spin motion). The high speed, longevity, contamination, and low loss requirements for these flywheels mandate that MB be used for suspension of the spinning rotor. The clearance of the MB and its catcher (backup) bearing is typically around 0.02 in (0.5 mm) and 0.01 in (0.25 mm), respectively, therefore, relative flywheel translational motions and rotational angles are expected to be very small. However, the feedback signal in the MB suspension system is the relative flywheel displacement respect to the reference body/flywheel housing; therefore, the relative translational motion of the flywheel with respect to the housing/casing should be considered. In contrast to the assumption employed in the previous work, the MB stiffness is intentionally set to a low value to yield high frequency force isolation between the satellite and the spinning shaft. The stiffness and damping of the MB may be conveniently adjusted through gain changes in the feedback control electronics.

2 Equations of Motion

The translational and rotational equations of motion for one gimbaled flywheel module can be derived in the flywheel housing frame and flywheel nonspinning body coordinate frame, respectively. The flywheel nonspinning coordinate has attractive merits such as the direction cosine matrix (DCM) between the flywheel and the housing can be obtained by two Euler sequential rotations rather than three in the flywheel nonspinning coordinate system, furthermore, the original modified Rodriguez parameters (MRP) are never singular due to the fact that the flywheel transverse angles are small. The satellite rotational motions have their coordinates expressed in the satellite body fixed frame and translational motions are not considered in this paper based on the assumptions that the displacement of the satellite mass center with respect to the system mass center is very small, the initial positions and velocities are zero, and there are no external forces. The gimbals are constrained to execute only precession.

2.1 Notation. Superscripts and subscripts in the vector expressions indicate that the coordinate frame and relative motion, respectively. The derivative of a vector with respect to the inertial reference frame will be denoted by

$$\frac{d}{dt}(X) = \dot{X}$$

and an overdot with parenthesis and subscript denotes that the differentiation with respect to time as viewed in the frame indicated by the subscript. The skew symmetric operator \( \tilde{X} \) is employed to represent a vector cross product, i.e., the cross product between vectors \( X \) and \( Y \) is \( X \times Y = \tilde{X} Y \), where \( \tilde{X} \) is defined as

$$\tilde{X} = \begin{bmatrix} 0 & -X_3 & X_2 \\ X_3 & 0 & -X_1 \\ -X_2 & X_1 & 0 \end{bmatrix} $$

For convenience, the vector identity for vector triple cross products is also employed to transform and simplify the terms into a numerically manageable form.

$$A \times (B \times C) = (A^T C) B - (A^T B) C = (A^T C I_{3\times3} - CA^T) B$$

Also define \( \text{Trieq}(A, C) = (A^T C I_{3\times3} - CA^T) \)

2.2 Flywheel Translational Motion (Flywheel Housing Coordinates). Figure 1 shows coordinate systems and one magnetically suspended, VSCMG module. The equations of the gimbaled flywheel’s absolute acceleration and translational motion in the flywheel housing frame are (1) and (2), respectively. The gimbal coordinates \((g_1, g_2, g_3)\) and flywheel housing coordinates \((h_1, h_2, h_3)\) are identical since each housing end is attached to the gimbal, therefore, no relative motion exists between the gimbal and flywheel housing \([GH]=I_{3\times3},[BG]=[BR]\) where \([GH]\) is the direction cosine matrix between the gimbal frame and the flywheel housing frame. The “\(x\)” variable in Eq. (2) is the relative displacement between the MB suspended flywheel and the flywheel housing coordinate system, which is identical with the gimbal coordinate system.

$$\dot{X}^{g}_{/h} = (\gamma + [BG]^T \Omega) \times 2 \dot{\gamma} + ([BG]^T \Omega \times \gamma) + [BG]^T \Omega \times x + (\dot{\gamma} + [BG]^T \Omega) \times x + [BG]^T \Omega + ([BG]^T \Omega \times [BG]^T R) + [BG]^T \Omega \times [BG]^T R$$

where

$$m\ddot{x} + a\dot{x} + b\dot{x} + c\Omega + d\Omega = F_f$$
2.3 Flywheel Rotational Motion (Flywheel Nonspinning Coordinates). As mentioned previously, the flywheel rotational motion is expressed with coordinates in the flywheel nonspinning frame \((\hat{f}_1, \hat{f}_2, \hat{f}_3)\) and is obtained from a Newton–Euler formulation. Equations (3) and (4) are the angular momentum vector and its inertial time derivative for the gimbaled flywheel module in terms of the flywheel rotational inertia and its absolute angular velocity.

\[
H_j = I_j \omega_j/n
\]  
(3)

\[
H_j' = H_j' + (\Omega \times H_j') \times H_j' = T_j
\]  
(4)

Without loss of generality, Eq. (4) can be rewritten as Eq. (5), which is suitable for numerical integration.

\[
\{ Q, [\Omega] \} \hat{t} + [Q_2, \Omega] \hat{t} + [Q_3, \Omega] \hat{t} + [Q_4, \Omega] = T_j
\]  
(5)

where \(I_j = \text{diag}[I_{la}, I_{lb}, I_c]\), \(\Omega = [\Omega_1, \Omega_2, \Omega_3]^T\), \(\omega_j/n = \Omega_f\)
\([Q_2] = [\Omega_2, \Omega_3, \Omega_1]^T\), \(\beta_1 = I_{lb} (\hat{\Omega}_2 - \hat{\Omega}_1), \beta_2 = I_{la} (\hat{\Omega}_3 - \hat{\Omega}_1), \beta_3 = I_{lb} (\hat{\Omega}_2 + \hat{\Omega}_3)\), \(\beta_4 = I_{lb} (\hat{\Omega}_1 + \hat{\Omega}_3)\), \(\beta_5 = I_{la} \Omega_2 + I_{lb} \Omega_3\), \(\beta_6 = I_{lb} \Omega_2 - I_{la} \Omega_3\), \(\beta_7 = I_{lb} \Omega_1 - I_{la} \Omega_3\), \(\beta_8 = I_{la} \Omega_2 + I_{lb} \Omega_3\), \(\beta_9 = I_{la} \Omega_2 - I_{lb} \Omega_3\), \(\beta_{10} = I_{lb} \Omega_1 + I_{la} \Omega_3\), \(\beta_{11} = I_{lb} \Omega_1 - I_{la} \Omega_3\), \(\beta_{12} = [\beta_1 I_1^T - \beta_2 I_2^T - \beta_3 I_3^T - \beta_4 I_4^T - \beta_5 I_5^T - \beta_6 I_6^T - \beta_7 I_7^T - \beta_8 I_8^T - \beta_9 I_9^T - \beta_{10} I_{10}^T - \beta_{11} I_{11}^T - \beta_{12} I_{12}^T]\), \(\beta_{13} = [\beta_1 I_1^T + \beta_2 I_2^T + \beta_3 I_3^T + \beta_4 I_4^T + \beta_5 I_5^T + \beta_6 I_6^T + \beta_7 I_7^T + \beta_8 I_8^T + \beta_9 I_9^T + \beta_{10} I_{10}^T + \beta_{11} I_{11}^T + \beta_{12} I_{12}^T]\), \(\beta_{14} = [\beta_1 I_1^T, I_2^T, I_3^T]^T\), \(\beta_{15} = [\beta_1 I_1^T, I_2^T, I_3^T]^T\)), \(\beta_{16} = [\beta_1 I_1^T, I_2^T, I_3^T]^T\)), \(\beta_{17} = [\beta_1 I_1^T, I_2^T, I_3^T]^T\)), \(\beta_{18} = [\beta_1 I_1^T, I_2^T, I_3^T]^T\)).

3 Nonlinear State Feedback Control and Power Tracking

The Lyapunov approach is employed to derive a nonlinear state feedback tracking control law, which is asymptotically stable for simultaneous attitude control and power tracking tasks, for rigid bearing models. Reference [19] introduced a logarithm term in the Lyapunov function, which makes the feedback controller linear in attitude and velocity errors. The reference motion in the present development is obtained from Refs. [23,25], which indicates that the optimal control for a rigid body, minimum-time maneuver is the bang-bang type control and if the bang-bang control excites significant vibration of the flexible degrees of freedom, it can be smoothed out by using the sharpness of the control switch represented by \(\alpha\) [25].

3.1 Reference Motion Design. A rigid body undergoes an arbitrary three-dimensional reorientation by rotating about a single principal axis. A near minimum-time control function (smoothed bang-bang control) for this single axis, rest to rest maneuver has the following mathematical description [25].

\[
\dot{\theta} = u = \pm u_{\text{max}} |\Delta t, t^, t|
\]  
(8)

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where $\Delta t = \alpha \tau$ and $\alpha$ controls the sharpness of the switches, where $\alpha = 0$ yields the bang-bang instantaneous torque switches and $\alpha = 0.25$ yields the smoothest member of the family [23]. Performing the integration of Eq. (8) with the positive sign selected and imposing rest to rest maneuver boundary conditions (at $t_0 = 0; \theta_0 = 0$, $\theta_{01} = 0$ and at $t_1 = t_f; \theta_{11} = \theta_f$, $\dot{\theta}_{1} = 0$), yields

$$u_{\text{max}} = \frac{4 \theta_f}{t_f} \frac{1}{1 - 2\alpha + 0.4\alpha^2 t_f^3}$$

(10)

Therefore, if the Euler’s principle axis of rotation is given as $l$, then the corresponding MRPs for orientation, angular velocity, and angular acceleration become

$$\sigma_r = l \tan(\theta/4), \quad \Omega_r = l \Omega, \quad \Omega_{1r} = l \Omega_1$$

(11)

### 3.2 State Feedback Control and Power Tracking

The logarithmic in the following candidate Lyapunov function was initially introduced by Tsiotras [18,19]. This innovation yielded the result that the feedback control law becomes linear in the orientation (MRP) error. The Lyapunov function is

$$V = \frac{1}{2} \delta \omega^T I_r \delta \omega + 2k_1 \log(1 + \delta \sigma^T \delta \sigma)$$

(12)

where $\delta \omega = \Omega - [BR] \Omega_r$. To assure stability one needs to verify that the first time derivative of $V$ is negative definite. The time derivative of the inertia matrix in Eq. (13) is due to the total inertia matrix $I_r$, containing a time varying gimbal inertia component with coordinates in the satellite frame.

$$\dot{V} = \delta \omega^T I_r \delta \omega + \frac{1}{2} \delta \sigma^T \delta \sigma + k_1 \delta \sigma$$

(13)

Substitute Eq. (7) into Eq. (13) and define the parenthesis term to be $-k_2 \delta \omega$. This yields the feedback control torques in the case of one VSCMG module and no external torque, as

$$L_e = [D] \gamma_1 + [B] \dot{\gamma}_1 + [BF] T_{\text{int}}$$

(14)

where $[D] = [D_1] - [R_1]$, $[R_1] = 0.5[I, \delta \omega [BG] + J, \delta \omega [BG]]$, $\delta \omega = [BG] \delta \omega_1 - [BG] \delta \omega_2$, $J = J_1 - J_2$, and $[D_2]$ is defined in Eq. (7) and $L_e$ is the required attitude control torque vector. The dimensions of each vector and matrix shown in Eq. (14) are $[D] = 3 \times 1$, $\gamma_1 = 1 \times 1$, $[B] = 3 \times 1$, $\gamma_1 = 1 \times 1$, $[BF] = 3 \times 1$, $T_{\text{int}} = 1 \times 1$, and $L_e = 3 \times 1$, respectively.

The time derivative of the inertia matrix in Eq. (13) is replaced by $[R_1]$ in Eq. (14) and the second term of Eq. (14) can be ignored because of the fact that the gimbal transverse moment of inertia $[B]$ is much smaller than the flywheel spin moment of inertia and since the gimbal acceleration $\gamma_1$ is also small.

The required attitude control torque constraint (14) for one VSCMG module can be extended to the case of four VSCMG modules with four gimbal rates and four flywheels motor torques, as

$$[D] \gamma_1 + [E] T_{\text{int}} = L_e$$

(15)

where $[D] = [D_1, D_2, D_3, D_4]$, $[E] = [[BG], [BG], [BG], [BG]]$, $T_{\text{int}} = [I_{\text{int}}, T_{\text{int}}, T_{\text{int}}, T_{\text{int}}]$, and $L_e = -\Omega I_r \Omega_1 \Omega_r + k_1 \delta \sigma + k_3 \delta \sigma - \delta \omega [BG] T_{\text{null}}$. The total kinetic energy stored in the flywheel and its time rate of change (power) can be written as

$$E_{FW} = \frac{1}{2} \Omega_{r}^T T_{\text{int}} \Omega_{r}$$

(16)

$$P_{FW} = \left[ \Omega_{r}^T T_{\text{int}} \right] [\gamma]$$

(17)

where $I_{\text{FW}} = \text{diag} [I_{a1}, I_{a2}, I_{a3}, I_{a4}]$ and $[\gamma] = \left[ \Omega_{r}, \Omega_{r}^2, \Omega_{r}^3, \Omega_{r}^4 \right]$. Therefore, from Eqs. (15) and (17)

$$G_1 \eta_1 = L_e$$

(19)

where $G_1$ is a $3 \times 8$ by $8 \times 1$ matrix, respectively. The general solution of Eq. (19) can be expressed as

$$\eta = G_1^T L_e + \eta_{\text{null}}$$

(20)

where $G_1^T$ is general inverse matrix of $G_1$, which is obtained from range space of $G_1$, and $\eta_{\text{null}}$ is a vector obtained from the null space of $G_1$ (i.e., $G_1 \eta_{\text{null}} = 0$). The required power control constraint equation can be written from Eq. (18) as

$$G_2 \eta = P_{FW}$$

(21)

where $G_2$ is a $1 \times 8$ by $8 \times 1$ matrix. The vector $\eta_{\text{null}}$ is obtained after substituting Eq. (20) into Eq. (21). Define the modified power as $P_{\text{mod}} = P_{FW} - G_2^T G_2 L_e = G_2 \eta_{\text{null}}$. Since the null vector $\eta_{\text{null}}$ is obtained from the null space of $G_1$, there exists a vector satisfying

$$\eta_{\text{null}} = P_{NW} v$$

(22)

where $P_N$ is the orthogonal projection onto null space of $G_1$ and has the properties of $P_N P_N = 1$ and $P_N = I_N - G_1^T G_1 = I_N - G_1^T G_1 (G_1 G_1^T)^{-1} G_1$. Substitute Eq. (22) into the modified power equation $P_{\text{mod}} = G_2 \eta_{\text{null}}$ to obtain $G_2 P_N v = P_{\text{null}}$. The minimum norm solution yields

$$v = (G_2 P_N)^T (G_2 P_N G_2^T)^{-1} P_{\text{null}} = P_N G_2^T (G_2 P_N G_2^T)^{-1} P_{\text{null}}$$

(23)

Therefore, the power transfer torque becomes

$$\eta_{\text{null}} = P_N G_2^T (G_2 P_N G_2^T)^{-1} P_{\text{null}}$$

(24)

By combining Eqs. (20) and (24), the combined attitude control and power transfer torque becomes

$$0 \leq t \leq \Delta t$$

for $t_f - \Delta t = t_2$

for $t_1 \leq t \leq t_f + \Delta t = t_2$

for $t_2 \leq t \leq t_f - \Delta t = t_3$

for $t_3 \leq t \leq t_f$
\[ \eta = G_1 L_z + P_s G_2 (G_s P_s G_2^2)^{-1} P_m \]  

(25)

The leading term on the right hand side of Eq. (25) is evaluated with a weighted pseudo inverse instead of the standard Moore–Penrose inverse because of the fact that ideally the VSCMGs should act like classical CMGs when operated away from the single-gimbal CMG singularity configuration [24]. Then

\[ G_i^* = W G_i^T (G_i W G_i^T)^{-1} \]

(26)

where \( W \) is a diagonal FW/CMG mode weighting matrix, \( W = \text{diag}(W_{F1}, \ldots, W_{F4}, W_{G1}, \ldots, W_{G4}) \) and \( W_{Fi} \), \( W_{Gi} \) are flywheel and CMG weights, respectively, associated with how closely the VSCMGs are desired to behave like regular flywheels or like CMGs. To achieve the desired VSCMGs’ performance, the weights are made dependent on the proximity to a single-gimbal CMG singularity [24,26]. To determine the weights \( W_{Fi} \), consider the minimum singular value \( \delta \) of \( G_i \), i.e.,

\[ \delta = \det(G_i G_i^*) \]  

(27)

Reference [16] discussed the advantages of this approach versus utilizing the minimum singular value of \( G_i \) [24].

As the gimbal angle approach a singular CMG configuration, the parameter \( \delta \) will go to zero and the flywheel weights are then defined to be Eq. (28)

\[ W_{Fi} = W_{Fi}^0 \exp(- \mu \delta) \]

(28)

where \( W_{Fi}^0 \) and \( \mu \) are positive scalars to be selected by the control designer. The CMG weightings \( W_{Gi} \) are constant [24], independent of satellite orientation, and are selected by the control designer to improve the controller performance.

4 Five Axes Magnetic Bearing Suspension System

Generally, a magnetic bearing suspension system includes position sensors, controllers, filters, power amplifiers, and magnetic bearing actuators. Figure 2 shows a typical flywheel/housing module with a five-axis MB suspension system [17]. The proportional, integral, derivative (PID) controller gains and filters determine the overall system performance if MB component saturation is avoided. In this section, the numerical modeling of each component is briefly explained.

4.1 Position Sensor. The two popular position sensors widely utilized in the MB suspension system are the optical sensor and eddy current sensor and the transfer function of these position sensors may be approximated by a linear first-order filter shown as Eq. (29).

\[ G_s(s) = \frac{\zeta}{\tau_s + 1} \]  

(29)

where \( \tau_s \) is time constant, which including the sensor bandwidth. The bandwidth is typically 5 KHz, so that an ideal (infinite bandwidth) position sensor model is employed with dc gain \( (\zeta) \) in the present work.

4.2 PID Control. Magnetic suspension control laws vary widely according to the particular applications. These include both plant based version such as H-infinity, quasi-resonant (QR), sliding mode, etc., or variations of basic PID control. A simple PID type control is described here for illustration purposes. Filter models are included to represent the natural roll-off of power amplifiers, sensors, and MB actuators. The parallel PID paths are shaped to suppress noise or prevent instability. Typically the transfer functions have a form similar with

\[ G_p(s) = \frac{1}{\tau_p s + 1}, \quad G_d(s) = \frac{1}{\tau_d s + 1}, \quad G_d(s) = \frac{s}{(\tau_d s + 1)^2} \]  

(30)

the time constants of \( \tau_p \) and \( \tau_d \) are selected to make the cut-off frequencies, \( f_c = 1/(2 \pi \tau_d) \), equal to 4 KHz and 2 KHz for the radial and axial paths, respectively in this work.

4.3 Programmed Filter Stages. Some commonly used programmed filter stages for magnetic suspension control contain low pass, lead/lag, and notch filters. The transfer function of each filter stage can be written as Eq. (31), respectively.

\[ G_{lp}(s) = \frac{1}{(\tau_l s + 1)^2}, \quad G_s(s) = \frac{k_s(s + z)}{(s + p)}, \quad G_{bw}(s) = \frac{s^2 + \omega_n^2}{s^2 + (\omega_n/Q)s + \omega_n^2} \]

(31)

The numerical examples presented employ a lag compensator of the form (31) with \( k_s = 1 \) and this stage performs similarly to an integrator to reduce steady state error without wind-up problems.

4.4 Power Amplifier. Power amplifiers convert control voltage to MB coil currents via an internal servo circuit. Pulse width modulated (PWM) servo amplifiers are commonly employed due
to low power consumption and accurate tracking for the commanded currents. A simplified feedback model of a servo amplifier including nonlinearities such as voltage and current saturation is shown in the Fig. 3. This is a first-order representation of the amplifier, including the current servo and the coil voltage, resistance and inductance. The coil voltage may become saturated due to high frequency noise and the coil inductance, which makes control of high frequency gyroscopic poles quite challenging. The closed loop system transfer function of a servo power amplifier may be represented in a simplified form as Eq. (32).

\[ G_{P}(s) = \frac{i_{PA}}{V_{out}} = \frac{K_{PA}}{L_{s} + (R + \chi K_{PA})} \]  

where \( K_{PA} \) is the proportional gain, \( \chi \) is an internal current sensor sensitivity, and \( L, R \) are the coil inductance and resistance, respectively.

The proportional gain and sensor sensitivity can be selected by matching the transfer function to a first-order filter considering the overall gain and bandwidth.

### 4.5 Magnetic Bearing Actuator

The forces produced by a MB actuator on the spinning flywheel shaft are nonlinear function of currents and the shaft’s relative displacement in the actuator clearance. A MB actuator for satellite application will most likely incorporate permanent magnets to supply a bias field to minimize ohmic losses. The equivalent magnetic circuit for these bearings is quite complex, thus to illustrate the IPAC methodology, we only consider the simpler electromagnetic biased, heteropolar MB. More complex bearing models, which also include eddy currents, fringing, and leakage effects are discussed in Ref. [27], Figure 4 shows one of the two axes for this type of MB actuator including coils, forces, and currents [17]. The total magnetic force produced along the axis shown is expressed in the approximate form

\[ F = F^{*} = \frac{B^{2}A_{p}}{\mu_{0}} - \frac{B^{2}A_{p}}{\mu_{0}} \]  

where the magnetic flux density is \( B \), cross section area is \( A_{p} \), and the magnetic field constant \( \mu_{0} \). This equation is derived from Ampere’s law and the conservation of flux, which has the form \( \Phi = B \cdot A \) [28] for all segments \( j \) of a closed-loop. Ampere’s law \( (\oint H dl = Ni) \) yields the approximate result \( H_{j}I_{j} + H_{j}I_{j} + 2H_{j}c = Ni. \) From Fig. 5 and with the flux intensity \( (H) \) converted to \( B/\mu \) in the linear range of \( B/H \) curve, the results are

\[ \frac{B}{\mu_{s}}I_{s} + \frac{B}{\mu_{r}}I_{r} + 2 \frac{B}{\mu_{o}}I_{o} = Ni \]  

(34)

\[ \frac{\Phi}{\mu_{A_{x}}}I_{x} + \frac{\Phi}{\mu_{A_{y}}}I_{y} + 2 \frac{\Phi}{\mu_{A_{z}}}I_{z} = R_{f} \Phi + R_{f} \Phi + 2R_{f} \Phi = Ni \]  

(35)

where the reluctances are defined by \( R_{f} = 1/(\mu_{A_{x}}) \).

For high performance magnetic materials \( \mu_{s} \gg \mu_{r} \gg \mu_{o} \), which implies that \( R_{f} \ll R_{r} \ll R_{o} \). An approximate form for the second term in the Eq. (35) becomes \( 2R_{f} \Phi = Ni \) and when combined with \( \Phi = B \cdot A \), yields

\[ B_{o} = \frac{Ni}{2R_{f}A_{o}} \]  

(36)

Substituting Eq. (36) into Eq. (33), yields the magnetic force expression in (37), where \( \vec{F} = \vec{F}_{x} + \vec{F}_{y} \) and \( \vec{F} = \vec{F}_{z} \).

\[ F = \frac{1}{4} N^{2} \mu_{o} A_{p} \left( \frac{(i_{x} + j)^{2}}{(c - x)^{2}} - \frac{(i_{x} - j)^{2}}{(c + x)^{2}} \right) \]  

(37)

The linearized form of this magnetic bearing force expression is (38), where the MB position stiffness \( (K_{p}) \) and current stiffness \( (K_{c}) \) are obtained by differentiating Eq. (37) with respect to the rotor displacement \( (x_{r}) \), and the control current \( (i_{c}) \), about the operating points, which are typically zero.

\[ F = K_{p}x_{r} + K_{c}i_{c} \]  

(38)

where \( K_{p} = [dF/dx_{r}]_{x_{r}=0} = N^{2} \mu_{o} A_{p} / c \) and \( K_{c} = [dF/di_{c}]_{x_{r}=0} = N^{2} \mu_{o} A_{p} / c \)

Figure 6 represents the components in a typical, active MB system. The overall system performance and characteristics are dominated by the PID control gains and filters if saturation does not occur.
5 Numerical Simulations

The numerical simulations presented below are provided to illustrate the effectiveness of IPAC with magnetically suspended, gimbaled VSCMGs. Figure 7 shows the feedback control paths for (a) determining flywheel motor torques and gimbal rates for IPAC service and for (b) determining magnetic bearing forces to magnetically support the flywheels.

The satellite attitude and power transfer information (feedbacks 1 and 2) are fed back to the motor control and the flywheel position signals (feedback 3) from the position sensors are fed back to the magnetic bearing control.

The overall configuration of the simulation model including VSCMG’s alignment angles and relative position to the satellite is presented in Fig. 8. Four VSCMGs are aligned in a standard pyramid shape similar to Refs. [16,24,26] and offset from the satellite mass center by $R_i$. Details of a flywheel/housing module containing axial and radial bearings are depicted in Fig. 9. The axial (thrust) magnetic bearing is typically integrated with one of the radial bearing to form a “combination” bearing. It is assumed that the MBs are aligned along the housing reference line (center line) resulting in the MB coordinate being identical with the flywheel housing coordinate in Fig. 9.
Table 1 lists the system parameter values for the numerical simulation including the angle $\theta$ in Fig. 8, which is measured from the satellite body axis ($\hat{b}_1-\hat{b}_2$) plane to the VSCMG’s gimbal axis.

The parameter $\mu$ in Eq. (28) and Table 1 is selected by the control designer to improve the controller performance. Tables 2–4 list magnetic bearing properties, MB feedback loop components, and feedback controller gains, respectively.

5.1 Satellite Attitude Maneuver and Power Transfer. This section presents the results of a satellite control and power transfer case study. Stability, robustness, power, and attitude tracking and unknown disturbance rejection are demonstrated for the magnetic suspension of the VSCMGs and the IPAC nonlinear, state feedback control. Robustness of the controllers is demonstrated with satellite initial attitude error, system modeling error, and flywheel mass imbalance. The two cases considered are as follows.

- Case 1: Satellite initial attitude error $\sigma(0)$ defined in the Table 1.
- Case 2: System modeling error (satellite moment of inertia is increased by 15%) and the flywheel mass imbalance is imposed on VSCMG module 2 with no initial attitude error.

5.1.1 Reference Motion Design. The reference motion described in the previous section is designed such that the satellite changes its orientation 90 deg about the Euler principal axis (EPA) of rotation from the initial attitude ([$\mathbf{s}_n(0)$]) to the final attitude ([$\mathbf{s}_n(0)$]) and starting and ending with zero angular velocities. The EPA is obtained as the eigenvector, which corresponds to the eigenvalue +1 of the direction cosine matrix ($[\mathbf{C}]$).

\[
\begin{bmatrix}
39.52 & 0.0524 & 0.9170 \\
0.8037 & 0.4636 & -0.3729 \\
-0.4447 & 0.8844 & 0.1410
\end{bmatrix},
\]

then the EPA and principal angle can be obtained as

\[
l = \begin{bmatrix} 0.6286 \\ 0.6809 \\ 0.3756 \end{bmatrix} \quad \text{and} \quad \Phi = \cos^{-1} \left( \frac{1}{2} (C_{11} + C_{22} + C_{33} - 1) \right)
\]

As mentioned before, the satellite reference attitude and angular velocity can be obtained by utilizing Eq. (39). Figure 10 shows the satellite reference motions including the satellite rotational angle (90 deg EPA rotation in 300 s).

5.1.2 Case 1: Satellite Initial Attitude Error. In general, the initial actual satellite orientation will differ from the reference (target) value. For example, the initial attitude error in this work is assumed to be $(-0.025, 0.0375, 0)^T$ in terms of modified Rodriguez parameter, which corresponds to a 10.3 deg principal rotation angle deviation from the reference motion. Figure 11 indicates the satellite maneuver motions with this initial attitude error. The final rotational angle is 90.0016 deg, which is identical with

Table 1 VSCMGs simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Numerical value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>54.75</td>
<td>(deg)</td>
</tr>
<tr>
<td>$\Omega(0)$</td>
<td>$[0.0 0.0]$</td>
<td>(rad/s)</td>
</tr>
<tr>
<td>$\sigma(0)$</td>
<td>$[-0.025 0.0375 0]^T [0.0 0]$</td>
<td>(rad/s)</td>
</tr>
<tr>
<td>$\gamma(0)$</td>
<td>$[45 -45 -45]$</td>
<td>(deg)</td>
</tr>
<tr>
<td>$\gamma(0)$</td>
<td>$[0.0 0.0]$</td>
<td>(rad/s)</td>
</tr>
<tr>
<td>$\Omega_f(0)$</td>
<td>$[2.2 2.2 2.2] \times 10^4$</td>
<td>(rpm)</td>
</tr>
<tr>
<td>$L_I$</td>
<td>[585, 585, 439]</td>
<td>(kg m$^2$)</td>
</tr>
<tr>
<td>$I_f$</td>
<td>[0.19, 0.27, 0.27]</td>
<td>(kg m$^2$)</td>
</tr>
<tr>
<td>$J$</td>
<td>[0.03, 0.036, 0.036]</td>
<td>(kg m$^2$)</td>
</tr>
<tr>
<td>$m$</td>
<td>26.3</td>
<td>(kg)</td>
</tr>
<tr>
<td>$R_p$</td>
<td>[2.2, 2, 2]</td>
<td>(m)</td>
</tr>
<tr>
<td>$W_{F_i}$</td>
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<td></td>
</tr>
<tr>
<td>$W_{G_i}$</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>$1 \times 10^{-10}$</td>
<td></td>
</tr>
</tbody>
</table>

*Indicates case 1.
*Indicates case 2.

Table 2 Magnetic bearing properties

<table>
<thead>
<tr>
<th>Magnetic bearing</th>
<th>Current stiffness $K_{cut}$ (N/A)</th>
<th>Position stiffness $K_{pos}$ (N/m)</th>
<th>Load capacity (N)</th>
<th>Location (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combo (radial)</td>
<td>40.92</td>
<td>$1.225 \times 10^6$</td>
<td>400</td>
<td>$r_{lab}=0.2$</td>
</tr>
<tr>
<td>Combo (axial)</td>
<td>83.6</td>
<td>$1.4 \times 10^6$</td>
<td>800</td>
<td>$r_{lab}=0.2$</td>
</tr>
<tr>
<td>Radial</td>
<td>38.7</td>
<td>$1.05 \times 10^6$</td>
<td>400</td>
<td>$r_{lab}=0.2$</td>
</tr>
</tbody>
</table>
Satellite angular velocity and attitude error vectors are shown in Fig. 12. The initial amplitudes of error vectors are reduced by 50% at about 10 s into the maneuver due to the nonlinear state feedback gain selection [23]. Figure 13 shows the gimbal angles and gimbal rates. The initial gimbal angles are shown in Table 1 and the maximum gimbal rates occur at the start of the simulation to compensate for the initial attitude errors. Figures 14 and 15 contain flywheels motor torques and spin and transverse velocities, respectively. As shown in these figures, the flywheel transverse velocities are very small; however, they contribute MB torques into each flywheel module. Flywheel displacements at the sensor position and MB forces are shown in Figs. 16 and 17, respectively. The simulated and target power transfer time histories are presented in Fig. 18, which indicates near error free tracking. The time history of the determinant in Eq. (28) are shown in Fig. 19.

5.1.3 Case 2: System Modeling Error and Flywheel Mass Imbalance Disturbance. The modeling uncertainty in this case is a 15% increase in the simulation model’s satellite rotational moment of inertia relative to the reference value given in Table 1 and employed for determining the target trajectory (Eq. (8)).

The disturbance is an imbalance load imposed at the flywheel mass center of module 2, in the form of an imbalance eccentricity of $1 \times 10^{-5}$ in. (0.00025 mm). Figures 20 and 21 provide satellite reference and actual motions, respectively. In these figures, angular velocity vectors are not the same scale due to the angular velocity error vectors, which is very small for the magnetic bearing gain structure employed. De-
Fig. 10  Satellite reference motions

Fig. 11  Satellite actual motions

Fig. 12  Velocity and attitude error vectors

Fig. 13  Gimbal motions

Fig. 14  Flywheel RPMs and motor torques

Fig. 15  Flywheel transverse velocities
Fig. 16  Flywheel displacements at sensor position

Fig. 17  Magnetic bearing forces

Fig. 18  Power transfer time histories

Fig. 19  Determinant and weights factor

Fig. 20  Satellite reference motions

Fig. 21  Satellite actual motions
amplitude to increase by a factor of 10, however, the absolute vibration severity levels were very small for both gain sets.

5.2 Magnetic Bearing Stiffness and Gimbal Axis Direction Effects. The preceding simulations utilized a high effective stiffness value for the magnetic bearing, as defined by

$$K_{\text{eff}} = -|K_{\text{prop}}| + G_{\text{sensor}} \times G_{\text{prop}} \times G_{\text{power amp}} \times G_{\text{actuator}}$$

where the gains correspond to the shaft relative position sensor, the proportional feedback, the power amplifier (current per voltage), and the actuator (force per current), respectively. This formula applies only at zero (dc) frequency, so it represents the true static stiffness if the controller does not include an integrator or a lead or lag stage with nonunity zero frequency gains. It may be desirable to lower the stiffness to achieve a greater degree of isolation between the flywheel and satellite body. A stiffness variation study was performed to show how this may affect the system response during the attitude tracking sequence utilized in the above simulations. The two cases listed in Table 5 were considered. Case 1 has a high bearing stiffness ($1.09 \times 10^7$ N/m) and case 2 has a low bearing stiffness ($1.03 \times 10^7$ N/m).

The results of this study indicated a small change in magnetic bearing force and a large increase in vibration due to lowering the magnetic bearing stiffness. Although the percent increase in vibration was large, the overall vibration severity was very small for both cases. Results for shaft motion at the bearing and for bearing force are provided in Figs. 30 and 31. The attitude tracking error was not significantly affected by the decrease in effective bearing stiffness between these two cases.

Simulation runs were made to determine the effects of gimbal axis uncertainty. The results showed that for the lower bearing stiffness $1.03 \times 10^7$ N/m case in Table 5, the response became unstable when the change in the gimbal axis direction (for module 1 in Fig. 8) was greater than approximately 0.6 deg (relative to the direction of the gimbal utilized to determine the control law). The results showed that for the higher bearing stiffness $1.07 \times 10^7$ N/m case in Table 5 the response became unstable when the change in the gimbal axis direction was greater than approximately 3.0 deg (relative to the direction of the gimbal utilized to determine the control law). This clearly illustrates a need for the uncertainty analysis described in Ref. [20]. The simulation results for the effects of gimbal axis uncertainty are provided in the Fig. 32.

6 Conclusions

The theory and example presented illustrate the methodology and potential effectiveness of an IPAC system utilizing magnetically suspended flywheels. The example demonstrated robustness and performance in the presence of an initial attitude error, system modeling error, and flywheel mass imbalance. The cases presented showed excellent attitude and power tracking with flywheel displacements far less than the magnetic bearing clearances and magnetic bearing forces well within their simulated capacities (400 N). Future work will focus on simulating an IPAC system installed on a flexible satellite model with magnetically supported flywheels. Beyond this, we seek an opportunity to conduct a hardware based demonstration of the IPAC approach presented here.

Nomenclature

- $C$: nominal air gap between rotor and bearing
- $F_f$: flywheel force vector
- $H_f$: flywheel angular momentum in the flywheel coordinates
- $H_f^\dot{}$: time derivative of flywheel angular momentum in the flywheel coordinates
- $H_s$: gimbal angular momentum in the gimbal coordinates
- $H_s$: satellite body angular momentum in the satellite coordinates
- $i$: control current from power amplifier
- $i_b$: bias current through the coil
- $i_c$: control current through the coil
- $I_{f_i}$: ith flywheel polar moment of inertia
- $I_f$: flywheel rotational inertia

<table>
<thead>
<tr>
<th>Table 5 Simulation properties for bearing stiffness study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor gain (V/m)</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Case 1</td>
</tr>
<tr>
<td>Case 2</td>
</tr>
</tbody>
</table>
$I_s = \text{satellite body rotational inertia including gimbal mass contribution in the satellite frame}$

$J = \text{gimbal rotational inertia}$

$K_L = \text{lead/lag compensator gain}$

$I_r = \text{flux path length of the rotor lamination stack}$

$m = \text{flywheel mass}$

$m_\text{g} = \text{gimbal mass}$

$N = \text{number of coil turns}$

$R_i = \text{housing position vector in the satellite body coordinates}$

$T_f = \text{flywheel torque vector}$

$T_{\text{ext}} = \text{external torque vector applied to the satellite}$

$T_{\text{mb}} = \text{flywheel torque vector due to MB reaction forces}$

$T_{\text{ms}} = \text{flywheel motor torque vector}$

$\tau_i = \text{i\textsuperscript{th} flywheel module motor torque}$

$V_{\text{coil}} = \text{power amplifier voltage across MB coil}$

$V_{\text{ctrl}} = \text{controller voltage signal after filter stages}$

$X = \text{satellite displacement vector in the inertia coordinate}$

$[AB] = \text{direction cosine matrix between } \hat{a}_i \text{ and } \hat{b}_i$

$I_i = \text{i\textsuperscript{th} column vector of } I_{3 \times 3}$

$[PQ]_i = \text{i\textsuperscript{th} column vector of matrix } [PQ]$

$\hat{b}_i = \text{unit vector of satellite body coordinates}$

$\hat{f}_i = \text{unit vector of flywheel coordinates}$

$\hat{g}_i = \text{unit vector of gimbal coordinates}$

$\hat{h}_i = \text{unit vector of housing coordinates}$

$\hat{n}_i = \text{unit vector of inertial coordinates}$

$\dot{\gamma}_i = \text{gimbal rate}$

$\dot{\gamma}_i = \text{i\textsuperscript{th} gimbal rate}$

$\Omega = \text{satellite angular velocity vector}$

$\Omega_f = \text{flywheel angular velocity vector in the flywheel coordinates}$

$\Omega^\text{ns} = \text{nonspinning flywheel angular velocity vector relative to the housing}$

$\Omega^f = \text{satellite angular velocity vector in the flywheel coordinates}$

$\Omega^f_i = \text{i\textsuperscript{th} flywheel module spin velocity}$

Fig. 30 Flywheel displacements at sensor position and magnetic bearing force (case 1)

Fig. 31 Flywheel displacements at sensor position and magnetic bearing force (case 2)
Fig. 32 The effect of gimbal axis uncertainty with bearing stiffness $1.03 \times 10^5$ N/m

\[
\Omega_f = \text{satellite angular velocity vector in the gimbal coordinate}
\]
\[
\Omega_s = \text{satellite reference angular velocity vector}
\]
\[
\sigma = \text{satellite attitude vector (modified Rodriguez parameter)}
\]
\[
\sigma_r = \text{satellite reference attitude vector (modified Rodriguez parameter)}
\]
\[
\tau_p = \text{time constant of proportional path transfer function}
\]
\[
\tau_i = \text{time constant of integral path transfer function}
\]
\[
\tau_d = \text{time constant of derivative path transfer function}
\]
\[
\tau_f = \text{time constant of low pass filter transfer function}
\]
\[
\omega_{np} = \text{the center frequency of the filter}
\]
\[
k_1, k_2 = \text{nonlinear feedback motor control gains}
\]

References


