Catcher Bearing Life Prediction Using a Rainflow Counting Approach

Jung Gu Lee
Alan Palazzolo¹

¹Corresponding author.

Active magnetic bearings (AMBs) are increasingly being used in industrial machines such as compressors, turbines, and generators since they cause only minor friction losses, are lubrication free, and can be operated adaptively to optimize machinery reliability and performance. Although advanced control algorithms provide high AMB reliability, CBs are still needed for power failure and bearing overload events [1–3].

Some publications provide simulation results for rotordynamic system response following a drop event onto catcher bearings. Most researchers, like Ishii and Kirk [4], have modeled the CB as a linear spring, damper and have sought to optimize the CB performance based on that model. Fumagalli et al. [5] and Fumagalli [6] studied the effect of air gap, friction coefficient, and CB damping on the impact dynamics and also conducted rotor drop tests. Cole et al. [7] studied the effects of bearing width, and inner race speed on the rotordynamic response. Sun et al. [8,9] proposed a nonlinear ball bearing model with thermal growth, providing a more accurate component model for the CB. Although not directly written on catcher bearings, Taktak [10] et al. determined that the friction coefficient decreases as the sliding interface temperature increases and that the shear stress due to sliding also decreased. Although not directly written on catcher bearings, Böhmer et al. [11] conducted experiments to determine the influence of heat generation in the contact zone and found that: (i) heat generated in the contact zone increased contact pressure, the size of contact zone, and the amount of sliding, and (ii) fatigue strength for rolling contact fatigue decreased as the temperature increases.

The large majority of catcher bearing drop cases in the literature involves horizontal machines, which exhibit only backward whirl motion. Forward whirl motion has been reported only in vertical machines. Cuprio et al. [12] conducted some drop tests in the flywheel application, and their results show the forward whirl motion. Ransom et al. [13] reported forward whirl motion in a vertical high speed motor-compressor rotor drop. According to Schmied [14], large imbalance causes forward whirl motion during rotor drop.

Some standards, such as API [15] specify an acceptable minimum number of drop occurrences, yet there is very few publications that address life prediction of CB’s, in terms of the number of drop occurrences before failure. API specifies that “The auxiliary bearing system shall be designed to survive at least two depletions from maximum continuous speed to zero speed with the normal aerodynamic braking and nominal process induced thrust load.” Sun [16] determines the fatigue life of ball bearing – catcher bearings, however a Lundberg-Palmgren formula is employed, which is strictly valid only for steady continuous loading. In comparison, the Rainflow approach presented here is valid for random loading and includes effects of shear stress due to rub between the rotor and inner race, which is neglected in Ref. [16].

The present paper’s simulation model includes CBs, and a horizontal rotor modeled with beam finite elements. Contact loads, Hertzian stresses, sub-surface shear stresses, and thermal growths in the rolling bearing components are calculated using a nonlinear ball bearing model including thermal growth during touchdown. The Rainflow counting method is applied to the sub-surface shear stress-time history in order to predict fatigue life of a CB, including thermal effects. This same approach is also applicable to any rolling element bearing subjected to random loading, e.g., a wind turbine drive train bearing. Parametric studies are provided to determine effects of bearing support stiffness and damping, friction coefficient, air gap distance, rotor speed, and static side load on CB life. In addition, examples of forward whirl response the associated model parameters are provided.

1 Introduction

Active magnetic bearings (AMBs) are increasingly being used in industrial machines such as compressor, turbines, and generators since they cause only minor friction losses, are lubrication free, and can be operated adaptively to optimize machinery reliability and performance. Although advanced control algorithms provide high AMB reliability, CBs are still needed for power failure and bearing overload events [1–3].

Some publications provide simulation results for rotordynamic system response following a drop event onto catcher bearings. Most researchers, like Ishii and Kirk [4], have modeled the CB as a linear spring, damper and have sought to optimize the CB performance based on that model. Fumagalli et al. [5] and Fumagalli [6] studied the effect of air gap, friction coefficient, and CB damping on the impact dynamics and also conducted rotor drop tests. Cole et al. [7] studied the effects of bearing width, and inner race speed on the rotordynamic response. Sun et al. [8,9] proposed a nonlinear ball bearing model with thermal growth, providing a more accurate component model for the CB. Although not directly written on catcher bearings, Taktak [10] et al. determined that the friction coefficient decreases as the sliding interface temperature increases and that the shear stress due to sliding also decreased. Although not directly written on catcher bearings, Böhmer et al. [11] conducted experiments to determine the influence of heat generation in the contact zone and found that: (i) heat generated in the contact zone increased contact pressure, the size of contact zone, and the amount of sliding, and (ii) fatigue strength for rolling contact fatigue decreased as the temperature increases.

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The bearing drag torque which depends on bearing type, external load, lubricant and operating speed is calculated by an empirical formula [17]. The drag torque due to external load $T_l$ is given by Ref. [17]

$$T_l = f_1 F_{ex} d_m$$

where $f_1$ is a factor depending upon bearing geometry and relative bearing load, $F_{ex}$ is the external force acting on the bearing, and $d_m$ is the mean of the inner and outer diameters.

Another drag torque due to lubricant and operating speed, $T_v$ is written by

$$Tv = \frac{10^{-7} f_2 v_m^{7/3} \rho^{3/2}}{160 \times 10^{-7} f_1 d_m} \quad v_m \geq 2000$$

$$Tv = \frac{10^{-7} f_2 v_m^{7/3} \rho^{3/2}}{160 \times 10^{-7} f_1 d_m} \quad v_m \leq 2000$$

where $v_m$ is given in centistokes and $n$ in revolutions per minute, and $f_2$ is a factor depending on type of bearing and method of lubrication. A bearing thermal model is developed assuming a uniform, radial direction heat flux, similar to Jorgensen and Shin [18]. Figure 1 shows the thermal nodes in the bearing components and equivalent heat transfer network. Thermal resistances corresponding for each component are defined in Table 1 [8].

The power conservation equation has the following form at each temperature node:

$$m C_p \frac{dT}{dt} = q_i - q_o$$

where the parameter $m$ is a lumped thermal mass, $C_p$ is the specific heat, and $q_i$ and $q_o$ are the heat flux in and out of the system. The thermal system temperatures are calculated, then the free
thermal expansions of the outer race, inner race and ball are obtained from:

outer race:  
\[ e_o = \frac{\xi}{3} (1 + \nu) \left[ r_e T_{Le} (r_e + r_h) + \Delta T_e (r_e + r_h) \right] \tag{5} \]

inner race:  
\[ e_i = \frac{\xi}{3} (1 + \nu) \left[ r_i T_{Li} (r_e + r_h) \right] \tag{6} \]

ball:  
\[ e_b = \xi r_b \Delta T_b \tag{7} \]

where \( \xi, \nu, \) and \( r \) are the thermal expansion coefficient, the Poisson's ratio, and radius of the respective bearing components, respectively. Subscript \( b, e, i, \) and \( s \) represent ball, outer race, housing, inner race, and shaft, respectively. The contact load due to thermal expansion is expressed by

\[ F_t = k e^{1.5} \tag{8} \]

and the inner race. Using a modified Palmgren formula including the effect of the thermal load, the drag torque due to external, \( T_i \), is given by Ref. [19] as

\[ T_i = f_i (F_{ex} + F_t) d_m \tag{9} \]

2.2 Nonlinear Ball Bearing Model. The nonlinear ball bearing model excludes tilt deflections and is similar to that in Ref. [8]. Bearing components deflect in the \( x, y, \) and \( z \) directions shown in Fig. 2 in response to the external force \( \{F\} \). The \( r-z \) plane passes through the center of a ball at an angle \( \phi \) referenced to the \( x \) axis. The inner race cross section at a ball location is loaded by the contact force vector \( \{Q\} \) at the groove center \( p \), which has a displacement vector \( \{u\} \), where \( \{Q\}^T = \{Q, Q_i\} \) and \( \{u\}^T = \{u_e, u_i, u_s\} \). The vectors for different reference points are related by a transformation matrix \( T \):

\[ \{u\} = T \{X\}, \quad \{Q\} = T \{f\} \tag{10} \]

where

\[ T = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \{X\}^T = [xyz] \tag{11} \]

and the vector \( \{f\} \) represents an equivalent force vector at the reference coordinate. The dynamic equations of motion for the inner race are given by

\[ m_i \ddot{\{X\}} = \{F\} + \sum_{j=1}^{n} T_{ij}^T \{Q\}_j \tag{12} \]

where \( n \) is the number of balls and \( m_i \) is the mass of inner race. The contact force vector \( \{Q\} \) contributed by a ball is expressed as

\[ \{Q\} = \begin{bmatrix} Q_r \\ Q_s \end{bmatrix} = \begin{bmatrix} -Q \cos \alpha \sin \gamma \\ -Q \sin \alpha \sin \gamma \end{bmatrix} \tag{13} \]
Fig. 5 Sub-surface Shear Stress Ratio $\tau_0/\tau_{max}$ versus Ellipse Axis Ratio [17]

Table 2 S-N Curve Parameters for AISI-52100

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>25</th>
<th>80</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$ (GPa/°C)</td>
<td>0.01</td>
<td>5.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_2$ (GPa)</td>
<td>2.47</td>
<td>2.20</td>
<td>1.84</td>
<td>1.60</td>
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</table>

Fig. 6 Load distribution in the inner race

Fig. 7 The S-N Curve [11]
where $Q_i$ is a contact force component and $x$ is the contact angle between a ball and the inner race. Let the vector $\{v\}$ be the displacement of a ball center. Then the equations of motion for an individual ball including centrifugal force $F_c$ becomes

$$m_b \begin{bmatrix} \ddot{v}_x \\ \ddot{v}_z \end{bmatrix} = \begin{bmatrix} Q_i \cos x_i - Q_e \cos x_e + F_c \\ Q_i \sin x_i - Q_e \sin x_e \end{bmatrix}$$

(14)

where the subscripts $i$, $e$ represents the inner and outer races, respectively and $m_b$ is the mass of a ball. The outer race is inserted into the bearing housing which is supported by spring and damper

![Diagram](image-url)

**Fig. 8** (a) Dimensions Diagram (in mm) and (b) Finite Element Model for Example System [26]

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Example rotor and bearing data</th>
</tr>
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<tbody>
<tr>
<td><strong>Rotor</strong></td>
<td><strong>Bearing</strong></td>
</tr>
<tr>
<td>Mass of the rotor</td>
<td>97.3 kg</td>
</tr>
<tr>
<td>Polar moment of inertia of the rotor</td>
<td>0.39 kg m²</td>
</tr>
<tr>
<td>Transverse moment of inertia of the rotor</td>
<td>2.82 kg m²</td>
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<tr>
<td>Air gap</td>
<td>300 μm</td>
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<tr>
<td>Inner diameter of sleeve</td>
<td>60.6 mm</td>
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<table>
<thead>
<tr>
<th>Table 4</th>
<th>Simulation cases</th>
</tr>
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<td>Support Stiffness (N/m)</td>
<td>Nominal Case</td>
</tr>
<tr>
<td>Support Damping (N-s/m)</td>
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<tr>
<td>Sliding friction coefficient</td>
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<tr>
<td>Kinetic friction coefficient</td>
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<tr>
<td>Air gap (mm)</td>
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<tr>
<td>Side load (N)</td>
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</tr>
<tr>
<td>Rotor speed (rpm)</td>
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</tr>
<tr>
<td>Initial Temperature (°C)</td>
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</tr>
</tbody>
</table>

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Fig. 9 Simulation result of Nominal case; (a) orbit plot, (b) Hertzian stress distribution, (c) Hertzian stress versus time, (d) Hertzian stress versus angle (e) Temperature versus time, and (f) Rainflow histogram
\[
(m_e + m_h) \begin{bmatrix} \ddot{x}_e \\ \ddot{y}_e \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n} [Q_e \cos \phi_{e,j}] \cos \phi_j \\ \sum_{j=1}^{n} [Q_e \cos \phi_{e,j}] \sin \phi_j \end{bmatrix} - C_i \begin{bmatrix} \ddot{x}_e \\ \ddot{y}_e \end{bmatrix} - K \begin{bmatrix} x_e \\ y_e \end{bmatrix}
\]

(15)

where \(m_e\) and \(m_h\) are the masses of the outer race and the housing, and \(x_e\) and \(y_e\) are the displacements of housing. Let \(w_r\) and \(u_t\) be the displacements of the outer and inner race groove centers (\(q\) and \(p\), respectively) in the radial direction, and then the displacements of the inner race groove center \(p\) and the ball center are geometrically related as shown in Fig. 4. The lengths \(l_{oi}\), \(l_{oe}\) represent the distance between the ball center and the groove centers under no external force, and the lengths \(l_i\), \(l_e\) the distances under external forces. Using the geometric relation between the displacements of the groove centers and ball center, the following equations are obtained:

\[
\tan \phi_i = \frac{l_{oi} \sin \phi_a + u_t - v_z}{l_{oi} \cos \phi_a + u_t + \phi_i - v_z}
\]

(16)

\[
\tan \phi_e = \frac{l_{oe} \sin \phi_a + v_z}{l_{oe} \cos \phi_a + v_z - \phi_e - w_r}
\]

(17)

\[
l_i = \varepsilon_b + \sqrt{(l_{oi} \cos \phi_a + u_t + \phi_i - v_z)^2 + (l_{oi} \sin \phi_a + u_t - v_z)^2}
\]

(18)

\[
l_e = \varepsilon_b + \sqrt{(l_{oe} \cos \phi_a + v_z - \phi_e - w_r)^2 + (l_{oe} \sin \phi_a + v_z)^2}
\]

(19)

where the \(\varepsilon\) terms indicate the thermal expansions as defined in Eqs. (5)–(7). The relative deflections \(\delta\) at the contacts are then

\[
\delta_i = l_i - l_{oi}
\]

(20)

\[
\delta_e = l_e - l_{oe}
\]

(21)

The point contact forces are obtained from the modified Hertzian formula [8]

\[
Q_i = k_i \delta_i^{3/2} \left( \frac{3}{2} \beta \delta_i + 1 \right)
\]

(22)

\[
Q_e = k_e \delta_e^{3/2} \left( \frac{3}{2} \beta \delta_e + 1 \right)
\]

(23)

where \(\beta\) is linearly related to the coefficient of restitution of materials engaged in contact and ranges from 0.08 to 0.32 s/m for steel bronze [8]. The corresponding Hertzian point contact stress on the surface and at the center of the elliptical contact area is

\[
\sigma_{max} = \frac{3Q_e}{2\pi d_{e1}d_{e2}}
\]

(24)

2.3 Flexible Rotor Model. The flexible rotor is modeled with Timoshenko beam elements including shear deformation. The equation of motion for a flexible rotor bearing system can be written as:

\[
[M]\ddot{q} + [C + \Omega G]\dot{q} + [K]q = [F]
\]

(25)

where \(M\) is the mass matrix, \(C\) is the damping matrix, \(G\) is the gyroscopic matrix, and \(K\) is the shaft stiffness matrix [20]. The vector \(q\) contains the nodal degrees of freedom, \(F\) is the load vector including the imbalance force and the nonlinear catcher bearing forces, and \(\Omega\) is the angular velocity of the rotor. Each beam node has four degrees of freedom, two translations and two rotations. Equation (25) is written in modal coordinates as:
where $\Phi$ is the modal matrix of the undamped, normal modes for the rotor and $\rho$ is a vector of modal coordinates.

### 2.4 Rotor Drop Simulation Model

Figure 3 shows the rotor drop and deep groove ball bearing, catcher bearing model. The frame of reference $O(X,Y)$ is fixed to the stationary machinery frame. The geometric centers of the rotor and bearing inner race are $O_r$ and $O_b$, respectively. $(x_r,y_r)$ is the location of $O_r$ and $(x_b,y_b)$ is the location of $O_b$ in the fixed frame of reference. The contact angle between the rotor and CB is

$$
\gamma = \tan^{-1}\left(\frac{y_r - y_b}{x_r - x_b}\right)
$$

The contact force coefficient $K_c$ which depends on the material property and geometry of the rotor and inner race is a factor in the nonlinear modified Hertzian contact force between the rotor and inner race. Palmgren [21] introduced the contact force coefficient for line contact

$$
K_c = \left[\frac{0.39^{10/9}}{I} \left(\frac{4(1 - \nu_1^2)}{E_1} + \frac{4(1 - \nu_2^2)}{E_2}\right)\right]^{-1}
$$
The normal force at the contact point between the spinning rotor and inner race is:

\[ F_n = K_c d n^2 + \frac{3}{2} b_0 d / C_{16}/C_{17} > c_r e_r / C_{20} c_r \]  

where \( n \) is \( 10/9 \) for line contact and \( c_r \) is the radial clearance between rotor and inner race, and \( e_r \) is the distance between the rotor and inner race and is defined as follows:

\[ e_r = \sqrt{(x_r - x_i)^2 + (y_r - y_i)^2} \quad (30) \]

For sliding contact between the rotor and inner race, the friction force (tangential force) is calculated by multiplying the friction coefficient by the normal force.

\[ F_t = \mu_d F_n \]  

(31)

where \( \mu_d \) is the kinetic coefficient of friction. The tangential velocities of the inner race and rotor at the contact point are calculated in order to identify rolling and sliding conditions.

\[ V_r = \dot{\theta}_r R_r - \dot{X}_r \sin \gamma - \dot{Y}_r \cos \gamma \]  

(32)

\[ V_i = \dot{\theta}_i R_i - \dot{X}_i \sin \gamma + \dot{Y}_i \cos \gamma \]  

(33)

A rolling condition is applied when the tangential velocity of the rotor is the same as that of inner race. This means that there is no slip at the contact point, and the friction force \( F_t \) is a static frictional force, which satisfies

\[ |F_t| \leq \mu_s F_n \]  

(34)

where \( \mu_s \) is the static friction coefficient. The sign of the slip force is determined by the sign of the relative velocity, i.e.,

\[ F_t = \text{sign} (V_r - V_i) \mu_d F_n \]  

(35)

The tangential friction forces for a rolling contact condition are obtained from the rotor and inner race, angular equilibrium equations by solving the following equations for the \( F_t \):

\[ I_p \ddot{\theta}_p = -(F_{t1} + F_{t2}) \cdot R_t \]  

(36)

\[ I_{p1} \ddot{\theta}_{11} = F_{t1} R_{b1} - T_{d1} \]  

(37)
where $I_p$, $I_{ph}$ are the polar moments of inertia of the rotor and inner race, respectively. $T_d$ is the drag torque. The subscripts 1 and 2 indicate catcher bearings 1 and 2.

3 Fatigue Damage

3.1 Shear Stress Acting on the Catcher Bearing. The failure of rolling bearings in surface fatigue caused by the concentrated contact force applied perpendicular to the surface, initiates at a location below the stressed surface. To determine the magnitude of the subsurface shear stress, Palmgren and Lundberg showed that the amplitude of the subsurface shear stress is related to the Hertzian stress and ellipse ratio. The detailed derivation is explained in Ref. [17]. The subsurface shear stress $\tau_0$ is calculated from

$$\frac{2\tau_0}{\sigma_{max}} = \sqrt{(2t-1)/t(t+1)}$$

(39)

where $t$ is an auxiliary parameter determined by elliptic contact region as shown Fig. 5, and $\sigma_{max}$ is defined in Eq. (24).

$$\frac{b}{a} = \sqrt{(t^2-1)(2t-1)}$$

(40)

The semimajor $a$ and semiminor $b$ axes of the projected elliptical area are calculated by Hertzian contact theory [17]. Harris [17] shows that the surface shear stress is very small compared with the normal stress in most rolling bearing applications, however, surface shear stress is very important for predicting fatigue life of a rolling element bearing. The surface shear stress is given by Ref. [17] as

$$\tau_{surface} = \mu \sigma$$

(41)

where $\mu$ is the friction coefficient between ball and races and $\sigma$ is the normal stress. The friction coefficient is typically from 0 to 0.3. In this paper, the friction coefficient between the ball and races is set equal to 0.2 [17].

3.2 Load and Stress Distribution in the Races. The load distribution along the inner race varies with contact point between the rotor and inner race as illustrated in Fig. 6. For the case that the radial rotor contact external force acts on the inner race of the bearing, an equivalent load distribution is expressed by

$$F(\theta) = k(\theta) F_{ex}$$

(42)

where $k(\theta)$ is the load distribution factor defined by Ref. [22].
thermal expansion. For catcher bearing applications, radial loads are applied to the bearing during a rotor drop event. This results in a clearance (loss of contact) over one half of the bearing—which is depicted as the unloaded state shown in Fig. 6. In contrast the opposite half of the bearing is loaded, and internal radial clearance in this side is assumed to be zero because of the large contact force. Under the assumption that \( e = 0 \) the equivalent radial load is distributed from \(-\pi/2\) to \( \pi/2\), referenced to the contact point as shown in Fig. 6. The sub-surface and surface shear stress are calculated at any point along the race utilizing the equivalent radial load distribution and Eq. (39) and Eq. (41).

3.3 Rainflow Cycle Counting Method. Parts of the CB are subjected to time varying stresses during the rotor drop occurrences. The Rainflow Cycle Counting Method [23], which was proposed by Dowling and Socie in 1982, is employed to predict the fatigue life of the CB which results from these stresses. The Rainflow method is used to identify stress cycles, that is, the stress range and mean stress for each cycle. The Appendix shows the procedure for the cycle counting method. Cumulative damage \( D \) and number of cycles \( N \) to failure are determined using a histogram of cycle ranges and Miner’s rule. Miner’s rule is expressed as follows. Failure is expected to occur if

\[
D = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \ldots = \sum\frac{n_i}{N_i} \geq 1
\]

where \( n_i \) is the number of applied cycles and \( N_i \) is the number of cycles to failure at a certain stress amplitude \( \tau_i \), respectively. In this study, the critical cumulative damage value of \( D \) is chosen to be 1 in Eq. (44) and the fatigue life is expressed as,

\[
\text{Life} = \frac{1}{\sum n_i/N_i}
\]

3.4 The S-N Curve. Raje and Sadeghi [24] showed that rolling fatigue is similar to torsional fatigue by applying the S-N curve for torsional fatigue to calculate bearing fatigue life. The S-N Curve including thermal effects shown in Fig. 7 is represented by

\[
N_f = \left( \frac{2\sigma}{\tau_{\text{eff}}} \right)^B
\]

where

\[
2\alpha = -\hat{C}_1 T + \hat{C}_2
\]

\[
\tau_{\text{eff}} = \tau_{\text{friction}} + \tau_{\text{normal}}
\]

and \( \hat{C}_1, \hat{C}_2 \) are constants and \( B \) is positive and related to the slope of the torsional S-N curve. These parameters are listed in Table 2 for bearing steel AISI-52100. From Ref. [17], \( \tau_{\text{friction}} \) and \( \tau_{\text{normal}} \) correspond with \( \tau_0 \) in Eq. (39) and \( \tau_{\text{surface}} \) in Eq. (41). As discussed in Sec. 3.1, the surface shear stress cannot be neglected for the calculation of fatigue life. The actual states of normal and tangential stress on the surface are highly complex, requiring an approximate form for practical computational modeling. Reference [25] shows that the shear stress including the surface shear stress

<table>
<thead>
<tr>
<th>Table 5 Life prediction summary</th>
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<tbody>
<tr>
<td>Temperature of inner race (°C)</td>
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<tr>
<td>Temperature of outer race (°C)</td>
</tr>
<tr>
<td>No. of drop occurrences to failure</td>
</tr>
<tr>
<td>Time to backward whirl motion cessation (sec)</td>
</tr>
</tbody>
</table>
rotor, which is depicted along with its FEM model in Fig. 8. The total of the race, the model evaluates life at a finite number of points. Although an infinite number of possible contact points exist along the races, and balls randomly change after the rotor drop. Therefore, an effective load effect as illustrated in Fig. 6 and described in Eqs. (41)–(42). The race contact force at some other location due to the effective load is determined by how long reverse whirl is sustained, and a one-second duration is adequate for a reverse whirl free drop event.

The example system consists of two CBs, and a horizontal rotor, which is depicted along with its FEM model in Fig. 8. The FEM rotor has 11 elements. The rotor and bearing specifications are listed in Tables 3. The model also includes flexible damped supports for the catcher bearings, which is typical for industrial applications. The stiffness and damping of these supports are 50,000,000 N/m and 5000 Ns/m, respectively. The dynamic coefficients of the support are assumed to be independent of rotor speed, and the values were obtained from Ref. [26]. Transient responses are obtained by utilizing Newmark Beta based numerical integration, with a time step of 1e-4s. The following parameters are varied to investigate their effect on fatigue life: (a) bearing support stiffness and damping, (b) friction coefficient, (c) side load due to an applied magnetic bearing, (d) air gap, and (e) rotor speed. The simulation cases are summarized in the Table 4. The duration of the numerical integration is determined by how long reverse whirl is sustained, and a one-second duration is adequate for a reverse whirl free drop event.

The contact point locations and load levels between the rotor, races, and balls randomly change after the rotor drop. Therefore the fatigue life varies around the circumferences of the races. Although an infinite number of possible contact points exist along the race, the model evaluates life at a finite number of points. A total of \( n \) equally spaced “test” points are located along the circumference of the inner race as shown in Fig. 6. Note that the stresses and damage at any test point may be affected by a rotor/race contact force at some other location due to the effective load effect as illustrated in Fig. 6 and described in Eqs. (41)–(42). The life is evaluated at each of the test points utilizing the Rainflow counting method and Miner’s rule. A study was conducted to determine an appropriate number of test points. For this case the rotor speed = 20,000 rpm, \( \mu_s = 0.3 \), \( \mu_f = 0.3 \) air gap = 0.3 mm and there is no side load. This case is included solely to illustrate the general response features and not for life evaluation, so its simulation duration is only one second.

Figures 9(a) and 9(b) show an orbit of the rotor at the bearing node and Hertzian contact stress distribution with the number of test points \( n = 100 \). The red solid circle shown in Fig. 9(a) indicates the unloaded clearance circle. The Hertzian contact stress time history at a test point and Hertzian contact stress distribution at an instant in time are shown in the Figs. 9(c) and 9(d), respectively. The number of cycles at each effective shear stress level, and each test point, are counted using the Rainflow counting algorithm. The fatigue life at each test point is then calculated from Eq. (45). The fatigue life is selected as that of the test point that has the maximum damage. The simulation results indicates that the damage and the fatigue life vary between test points, and the bearing life approaches a constant value as the number of test point increases as shown in Fig. 10. All of the following results were obtained utilizing 100 test points. Backward whirl motion occurs during rotor drop, induces high contact forces between the rotor and inner race, and may causes significant damage to the catcher bearing until the backward whirl motion diminishes. For the nominal case, the backward whirl motion diminishes after about 12s. Figure 10(c) indicates that the number of drop occurrence to failure sharply reduces while backward whirl motion is occurring.

4 Simulation Results and Discussion

The example system consists of two CBs, and a horizontal rotor, which is depicted along with its FEM model in Fig. 8. The FEM rotor has 11 elements. The rotor and bearing specifications are listed in Tables 3. The model also includes flexible damped supports for the catcher bearings, which is typical for industrial applications. The stiffness and damping of these supports are 50,000,000 N/m and 5000 Ns/m, respectively. The dynamic coefficients of the support are assumed to be independent of rotor speed, and the values were obtained from Ref. [26]. Transient responses are obtained by utilizing Newmark Beta based numerical integration, with a time step of 1e-4s. The following parameters are varied to investigate their effect on fatigue life: (a) bearing support stiffness and damping, (b) friction coefficient, (c) side load due to an applied magnetic bearing, (d) air gap, and (e) rotor speed. The simulation cases are summarized in the Table 4. The duration of the numerical integration is determined by how long reverse whirl is sustained, and a one-second duration is adequate for a reverse whirl free drop event.

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and the life prediction increases from 4 for the nominal case to 340 for the higher damping case as shown in Table 5.

4.2 Journal – Inner Race Contact Friction Effect. Table 4 – Case 3 considers a drop in static friction by a factor of 3 and in kinetic friction by a factor of 2. This causes the inner race to accelerate much slower as shown in Fig. 14. Comparisons of Fig. 11(a) versus 11(d), Fig. 12(a) versus 12(d), and Fig. 13(a) versus 13(d) result in the following conclusions for the case of reduced friction. The orbits show an elimination of backward whirl, the cycles of peak stress amplitudes (>1.0 GPa) are negligible as shown in Fig. 13’s Rainflow Histogram, and the life prediction increases from 4 for the nominal case to 82,000 for the reduced friction case as shown in Table 5.

4.3 Catcher Bearing Air Gap Size Effect. Comparisons of Fig. 11(a) versus 11(e), Fig. 12(a) versus 12(e), and Fig. 13(a) versus 13(e) result in the following conclusions for the case of increasing the air gap by 66%. The orbits show a strong backward whirl motion and a large increase in peak motion, the cycles of peak stress amplitudes above 1.5 GPa slightly increase as shown in Fig. 14’s Rainflow Histogram, and the life prediction decreases from 4 for the nominal case to 2 for the larger clearance case as shown in Table 5. Like case 1, the duration of backward whirl motion decreases by 75% due to higher thermal load.

4.4 Applied Side Load Effect. Comparisons of Fig. 11(a) versus 11(f), Fig. 12(a) versus 12(f), and Fig. 13(a) versus 13(f) result in the following conclusions for the case of applying a 500 N side load at each magnetic bearing. This type of event may occur, e.g., during controller tuning if control is accidentally lost due to instability at high speed, so that magnetic bearing power is still available to apply the side loads for mitigating the vibrations of the rotor on the catcher bearings. The orbits show an elimination of backward whirl, the peak contact force decreases by a factor of 6, the cycles of peak stress amplitudes (>1.0 GPa) are negligible as shown in Fig. 14’s Rainflow Histogram, and the life prediction increases from 4 for the nominal case to 8200 for the applied side load case as shown in Table 5.
4.5 Rotor Drop Speed (rpm) Effect. Comparisons of Fig. 11(a) versus 11(g), Fig. 12(a) versus 12(g), and Fig. 13(a) versus 13(g) result in the following conclusions for the case of decreasing the drop speed by 50%. The orbits show a weak backward whirl motion, the peak contact force decreases by 40%, the cycles of stress amplitudes through the whole range significantly decrease as shown in Fig. 14’s Rainflow Histogram, and the life prediction increases from 4 for the nominal case to 30 for the lower speed case as shown in Table 5. The 8X significant increase in the no. of drop occurrence to failure is due to the shorter duration of backward whirl motion and lower temperature increase in the CB. Increasing the rotor drop speed increases the temperature in the CB and reduces the fatigue life of the CB as shown in Fig. 15.

4.6 Forward Whirl Response. Figure 16 shows a comparison between light imbalance \(6 \times 10^{-5} \text{ kg m}\) and heavy imbalance \(6 \times 10^{-3} \text{ kg m}\), loading with a rotor speed of 10,000 rpm, \( \mu_c = 0.35 \), \( \mu_d = 0.3 \), an air gap = 0.3 mm and no side load. For the light imbalance case (a), the motion is relatively benign, with a slight oscillation in whirl rate. The heavy imbalance case in Fig. 16(b) exhibits a strong forward whirl as indicated by the whirl rate versus time plot. The number of drops events to failure in this case is eight times indicating that forward whirl can also be very destructive.

5 Conclusions

This paper employed a novel, high fidelity, thermal-structural, fully nonlinear ball bearing, and flexible finite element shaft model, and Rainflow counting approach to evaluate the life of catcher bearings in terms of number of drop occurrences to failure. The life prediction involved determining contact load, Hertzian stresses, subshesh stress, surface shear stress, and thermal growths. It was found that decreasing rotor-inner race contact friction, reducing catcher bearing air gap, applying a constant side load after a drop event, reducing support stiffness and increasing support damping, and reducing speed (rpm) all increase the life of an AMB catcher bearing. Backward whirl motion occurs in cases: nominal case, #1, #4, and #6. As shown in Fig. 17, the rotor speed sharply reduced due to backward whirl motion induced, friction torques without heat generation in the bearing components and between the rotor and inner race.

In addition, simulation results indicated that forward whirl can occur for a rotor with large imbalance. The paper represents the first effort to predict catcher bearing life based on actual transient stresses, unlike prior approaches that utilized empirical life relationships derived from constant loading tests. The number of predicted drops to failure, and its trend with regards to parameter changes, are consistent with test results reported in the literature [14]. Future work includes experimental verification and developing a similar analysis for roller and plain sleeve type catcher bearings.

Acknowledgment

The authors gratefully acknowledge support of this research by the Texas A&M Industrial Turbomachinery Research Consortium (TRC).

Appendix: Rainflow Cycle Counting Method

The Rainflow cycle counting is explained according to the ASTM E-1049 Standard Practices for Cycle Counting in Fatigue Analysis. Rules for the Rainflow counting method are given as follows:

(a) Read next peak or trough. If out of data, go to step (f).

### Table 6 Stress Cycle Count

<table>
<thead>
<tr>
<th>Stress range</th>
<th>Cycle counts</th>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
<td>D-G</td>
</tr>
<tr>
<td>9</td>
<td>0.5</td>
<td>C-D, G-H</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
<td>H-I</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
<td>B-C, E-F</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>A-B</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 17 Angular velocity of rotor versus Time for the backward whirl cases

Fig. 18 Stress versus time

![Stress versus time](image_url)
(b) If there are less than three points, go to step (a). Form ranges X and Y using the three most recent peaks and trough that have not been discarded.

(c) Compare the absolute values of ranges X and Y.
   (i) If X<Y, go to step (a).
   (ii) If X>Y, go to step (d).

(d) If range Y contains the starting point S, go to step (e); otherwise, count range Y as one cycle; discard the peak and trough of Y and go to (b).

(e) Count range Y as one-half cycle; discard the first point in range Y; move the starting point to the second point in range Y and go to (b).

(f) Count each range that has not been previously counted as one-half cycle.

The number of cycles corresponding to stress range illustrated in Fig. 18 is summarized in Table 6.

References


